Testing Linearity against Non-Signaling Strategies

Alessandro Chiesa **Peter Manohar** Igor Shinkar

UC Berkeley

What is linearity testing?

Linearity Testing

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Given oracle access to $f:\{0,1\}^n \rightarrow \{0,1\}$ decide if:

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(2) f is far from all linear functions.

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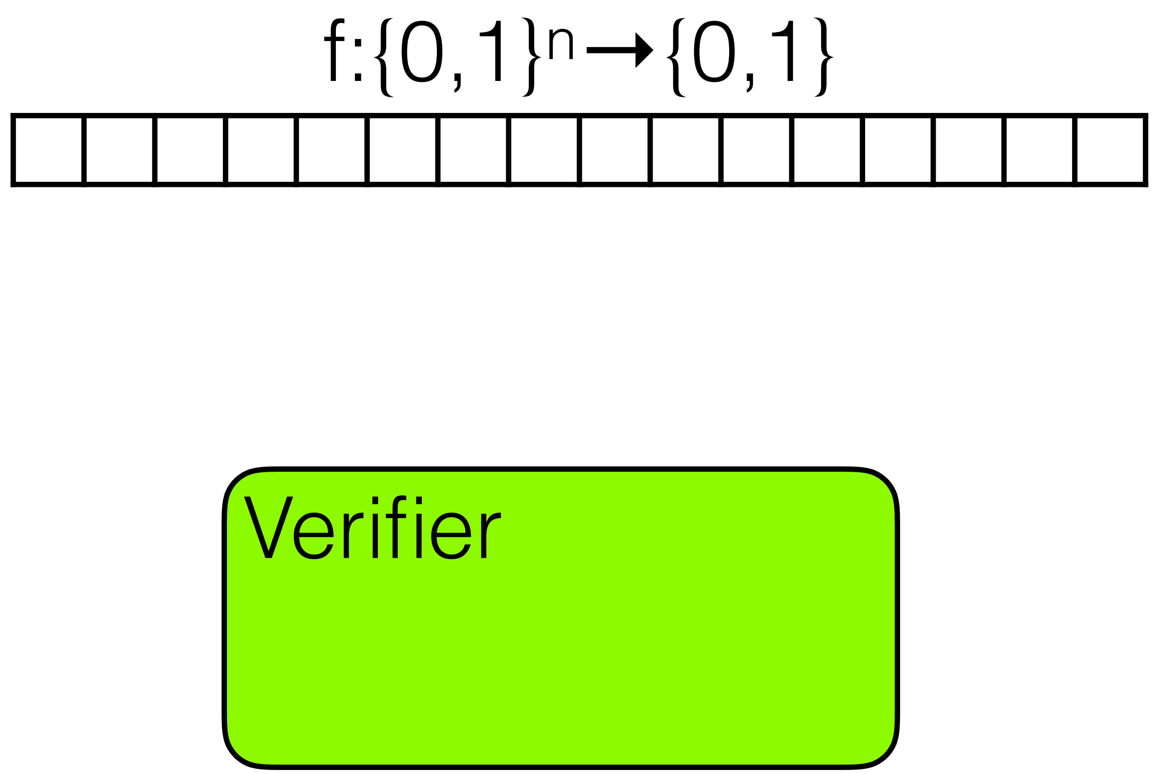

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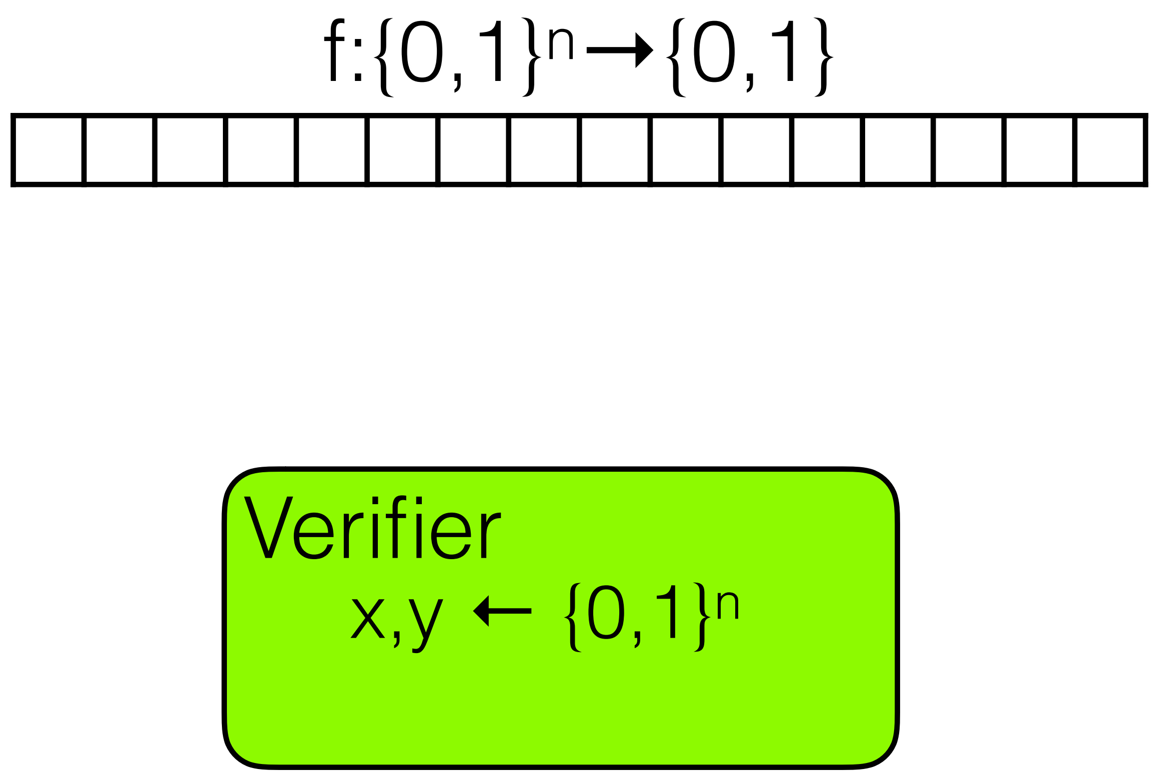

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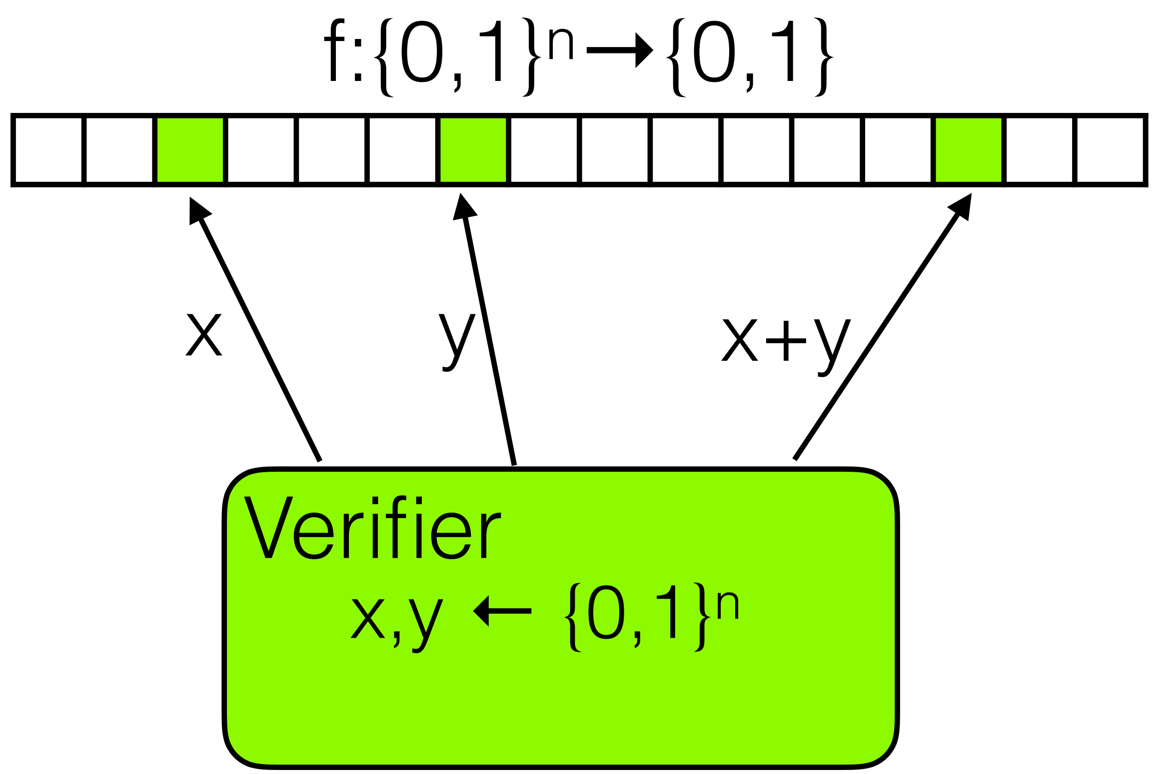

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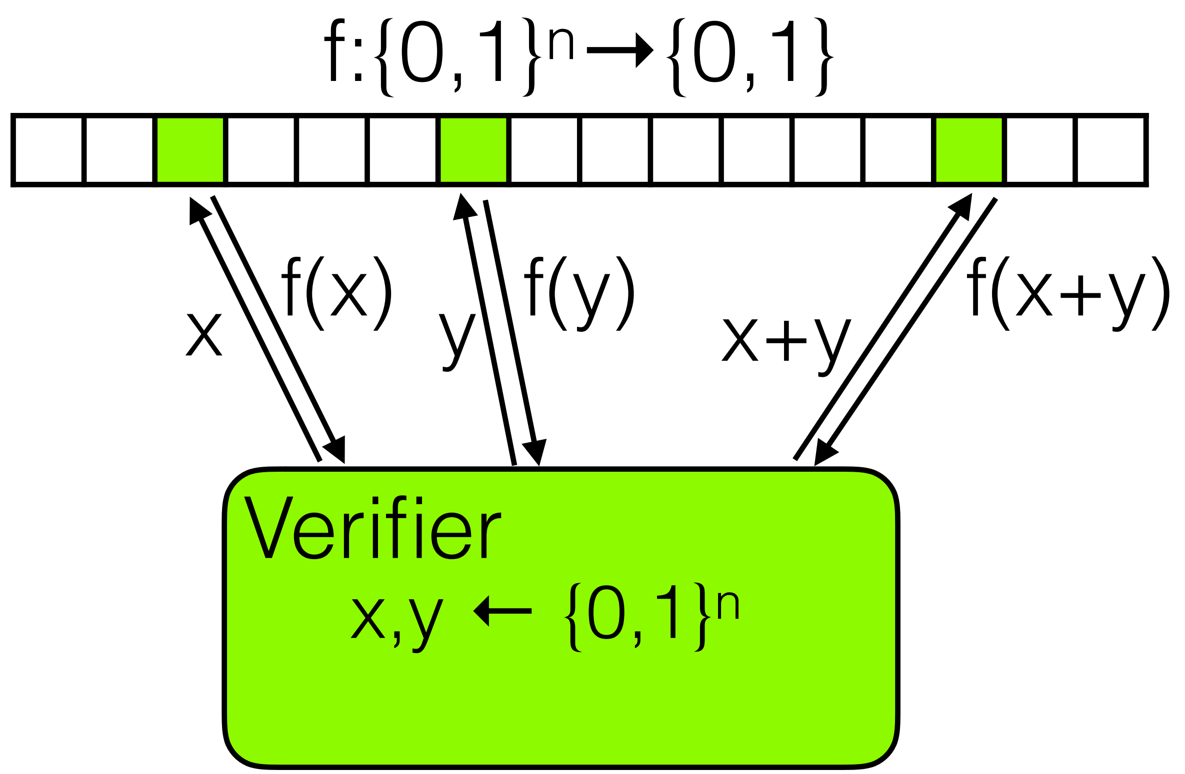

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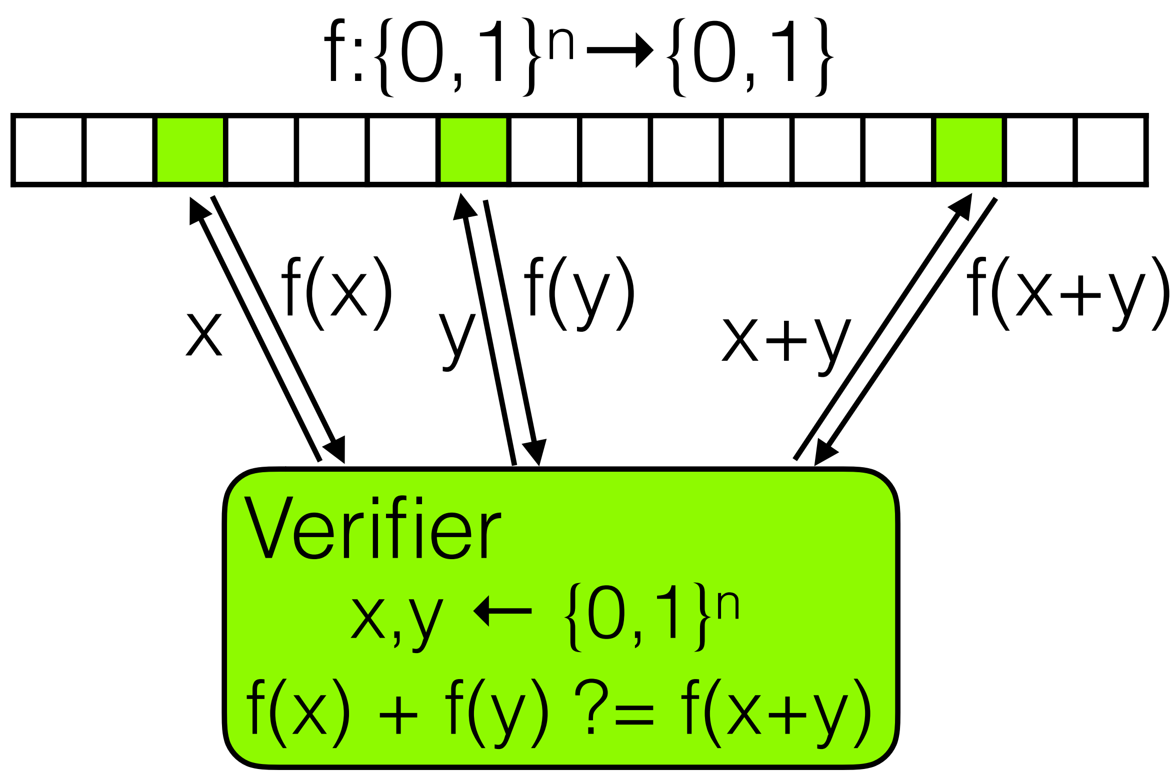

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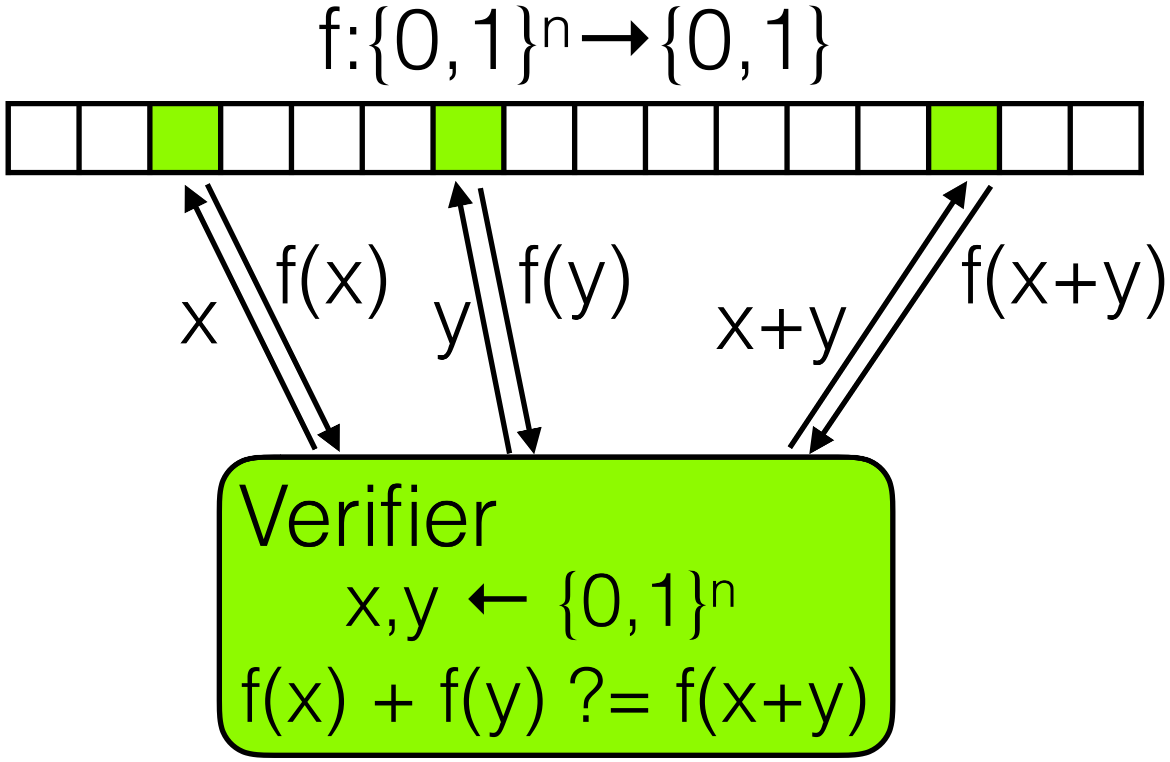

The test works:

$\Pr_{x,y}[\text{f passes}] \geq 1 - \varepsilon \rightarrow \Delta(f, \text{LIN}) \leq \varepsilon$ [BLR93, BCHKS96]

What are non-signaling strategies?

Non-Signaling Strategies

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Multiplayer games:

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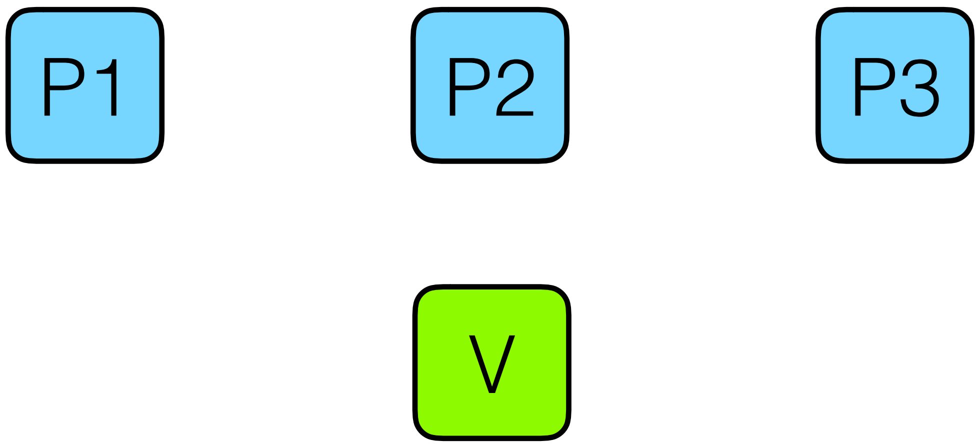

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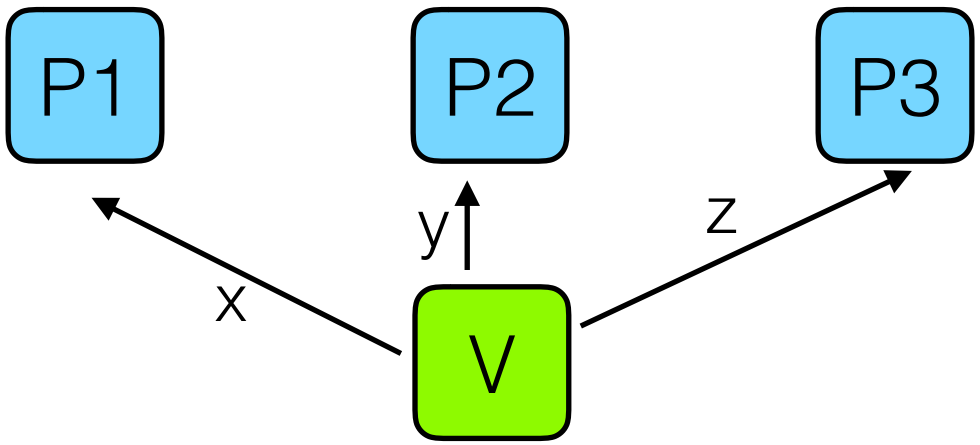

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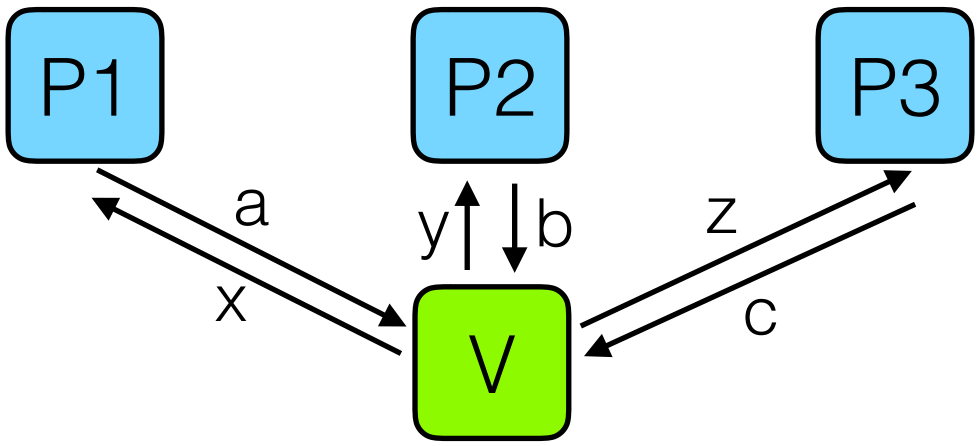

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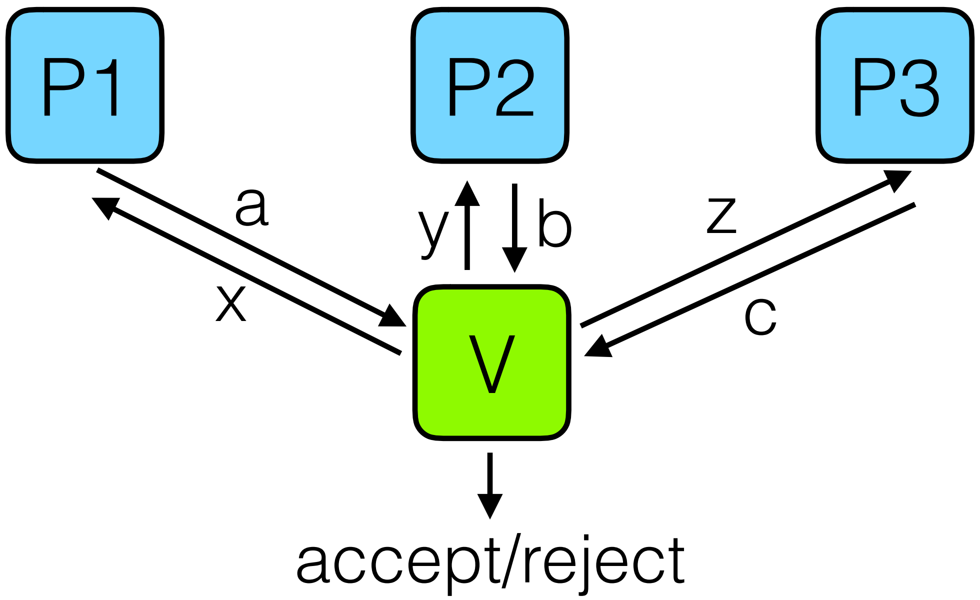

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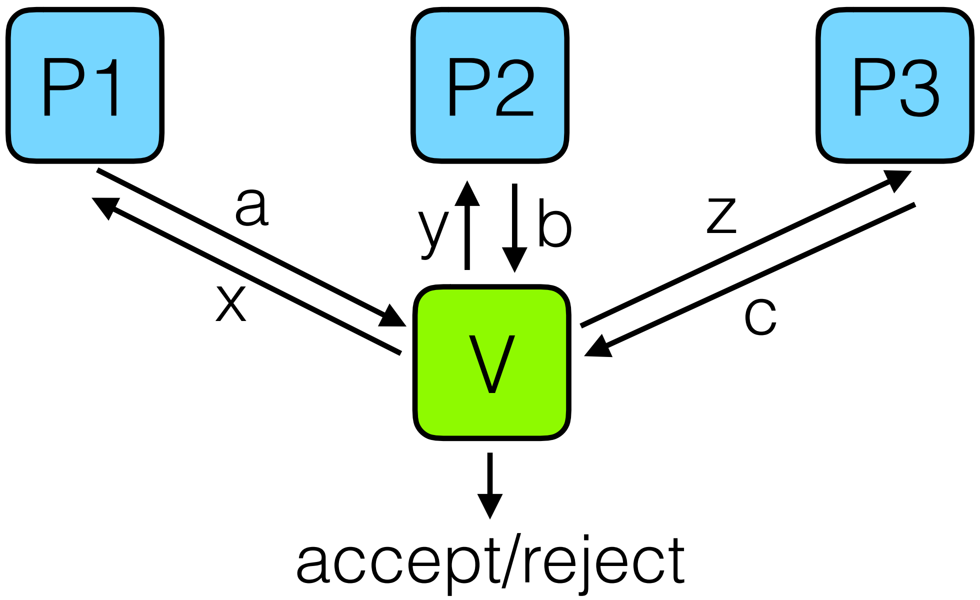

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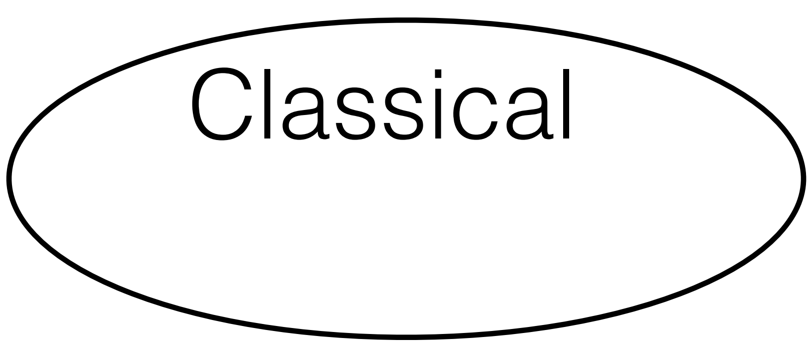

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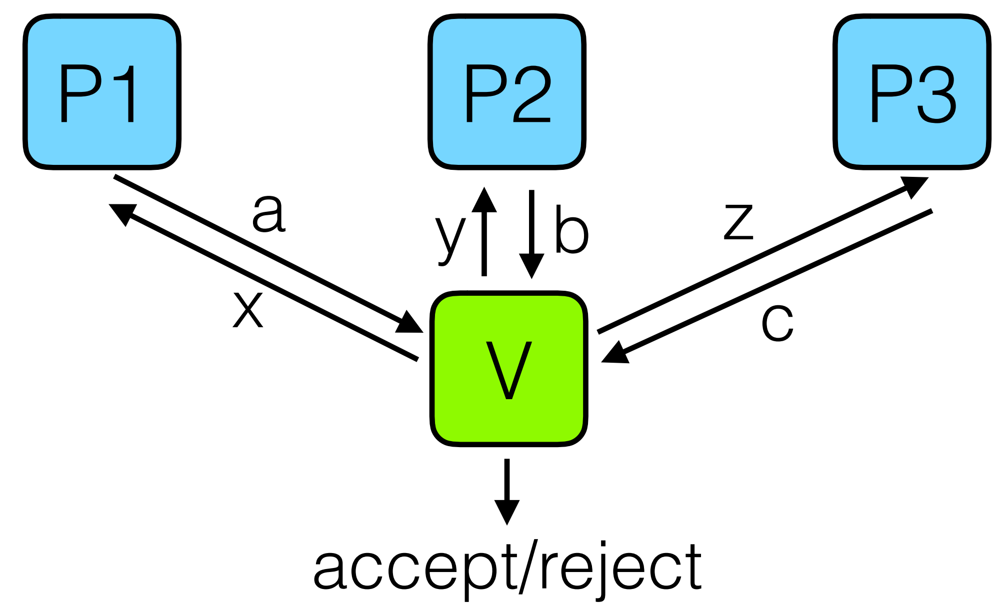

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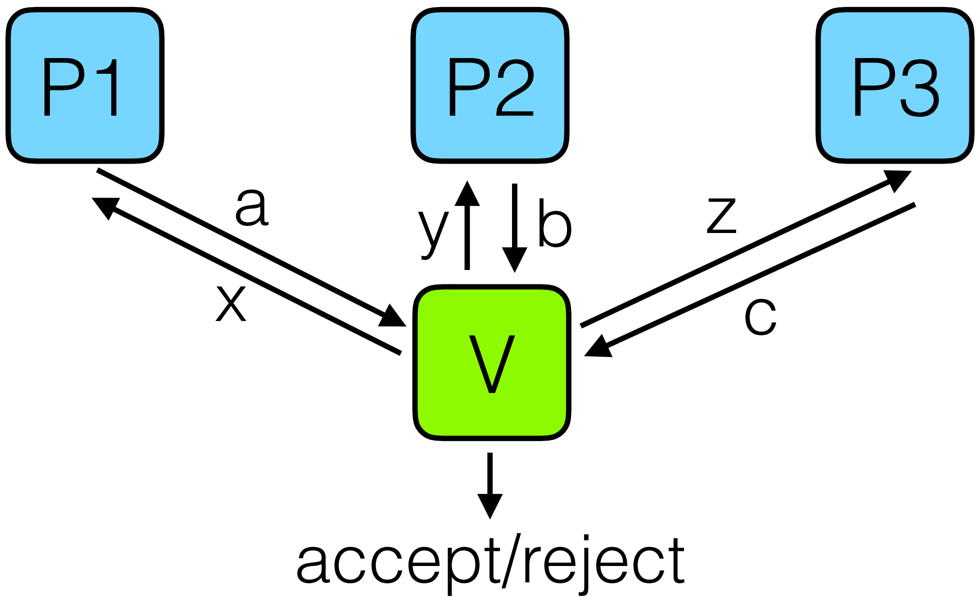

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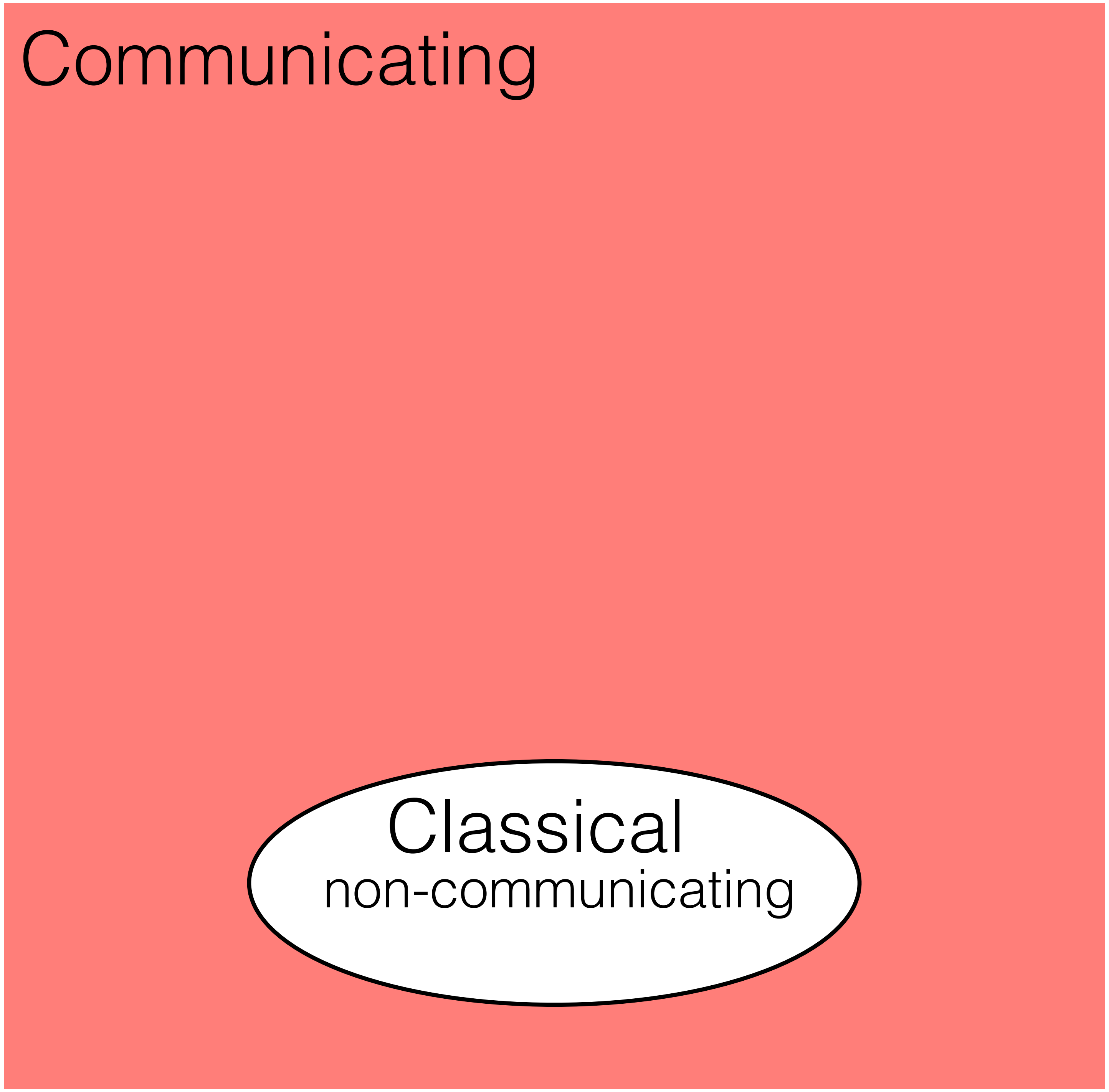

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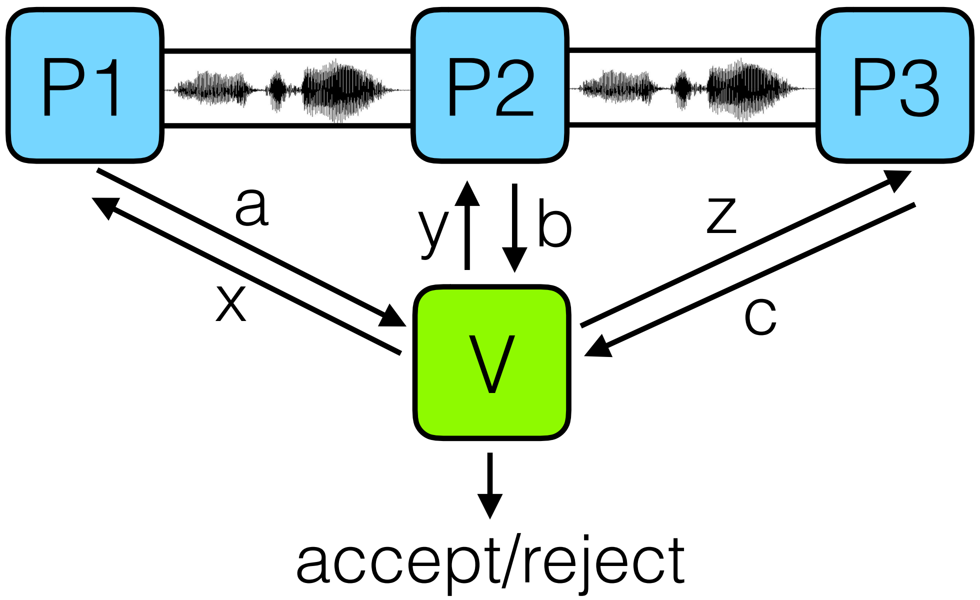

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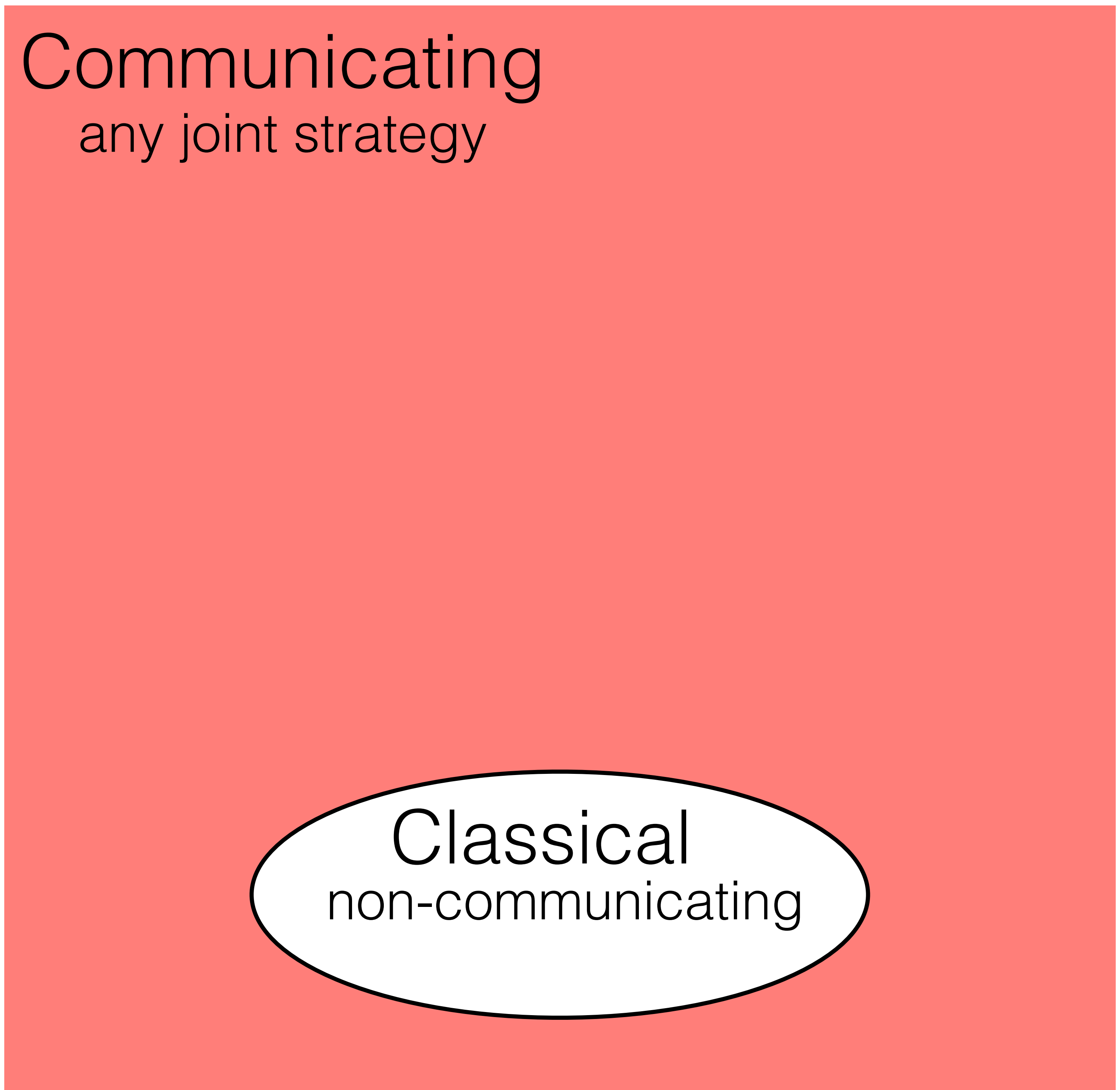

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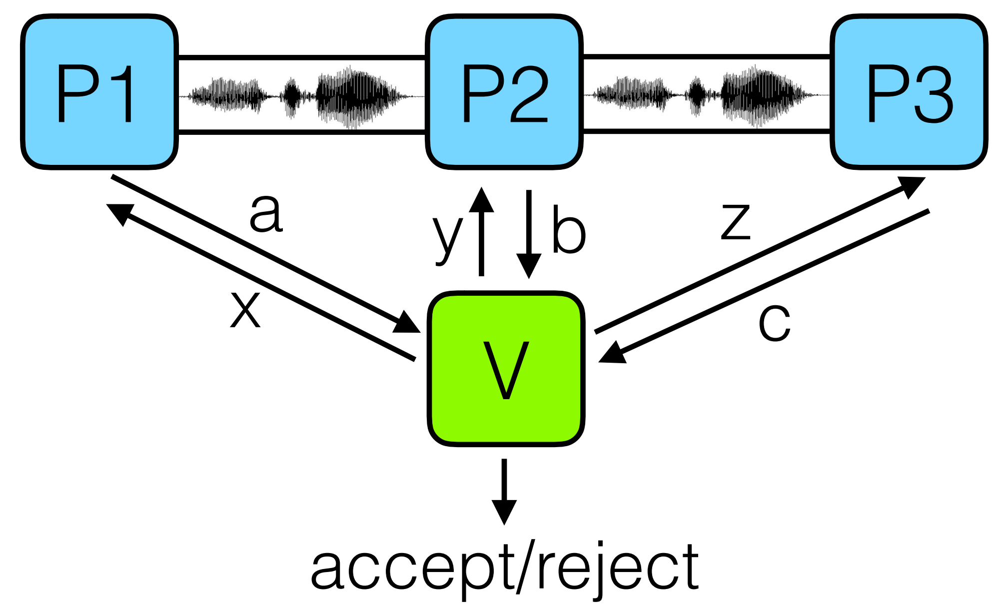

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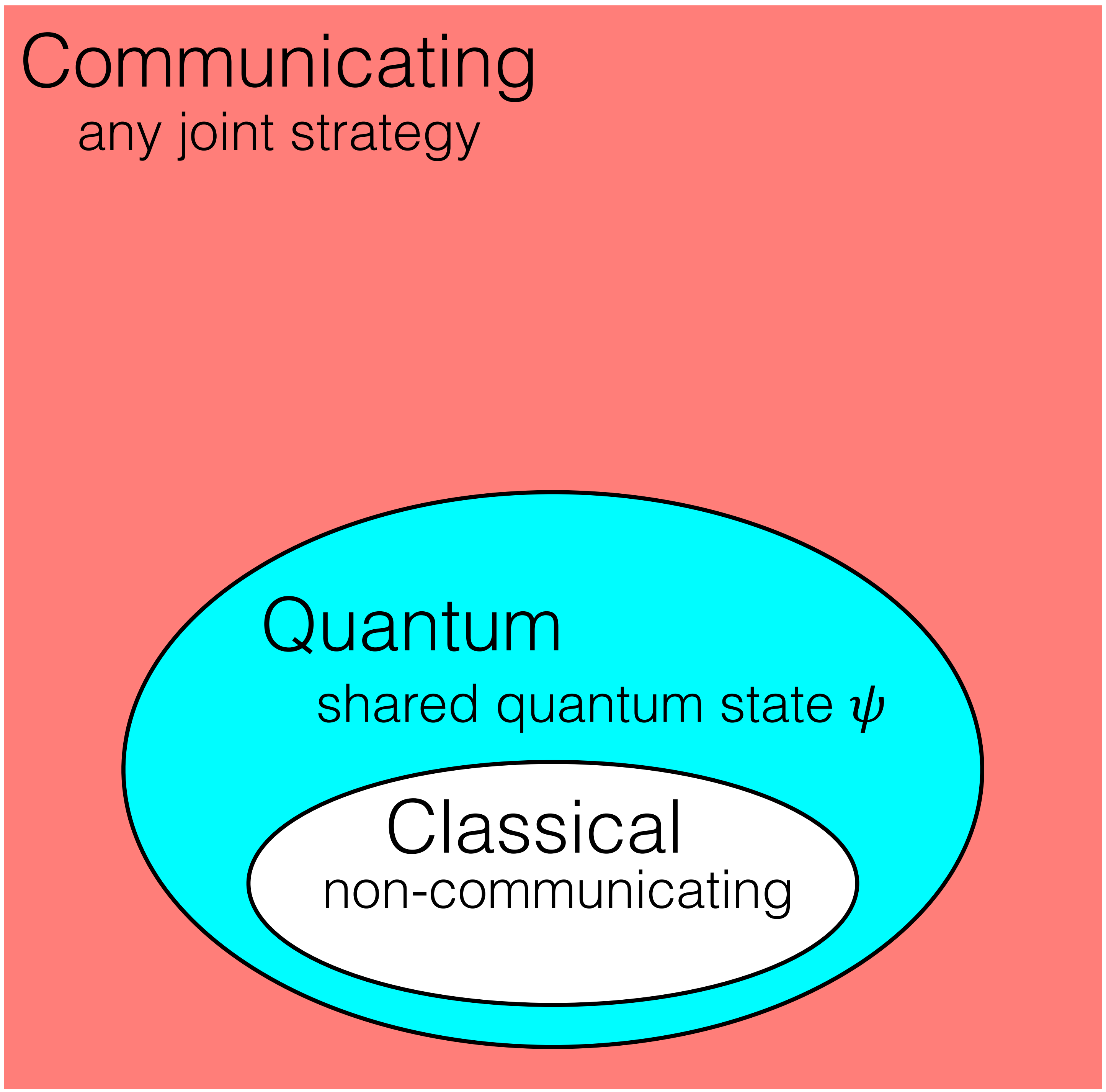

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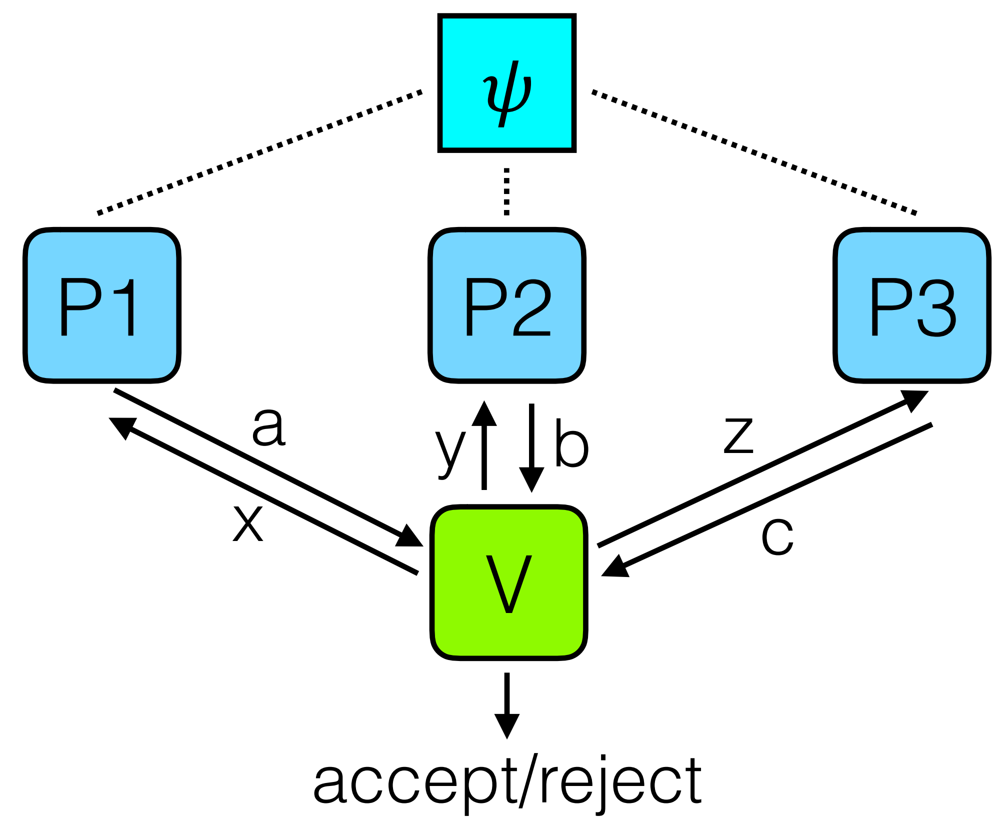

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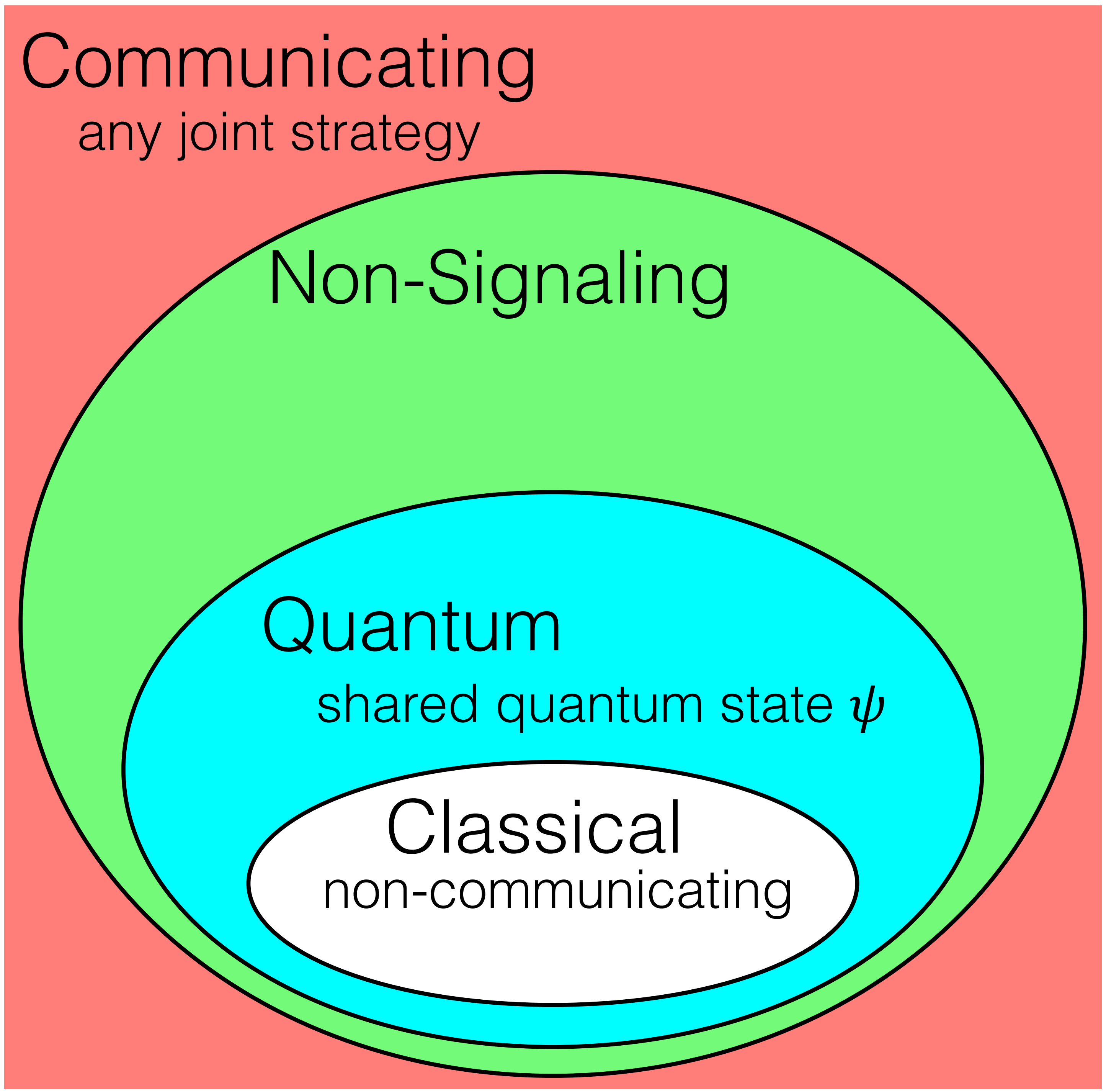

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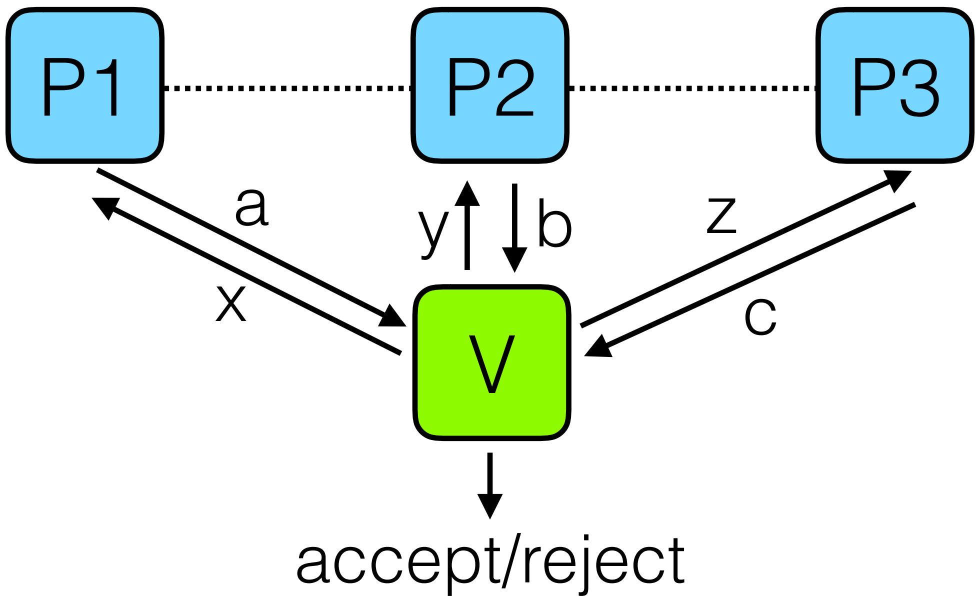

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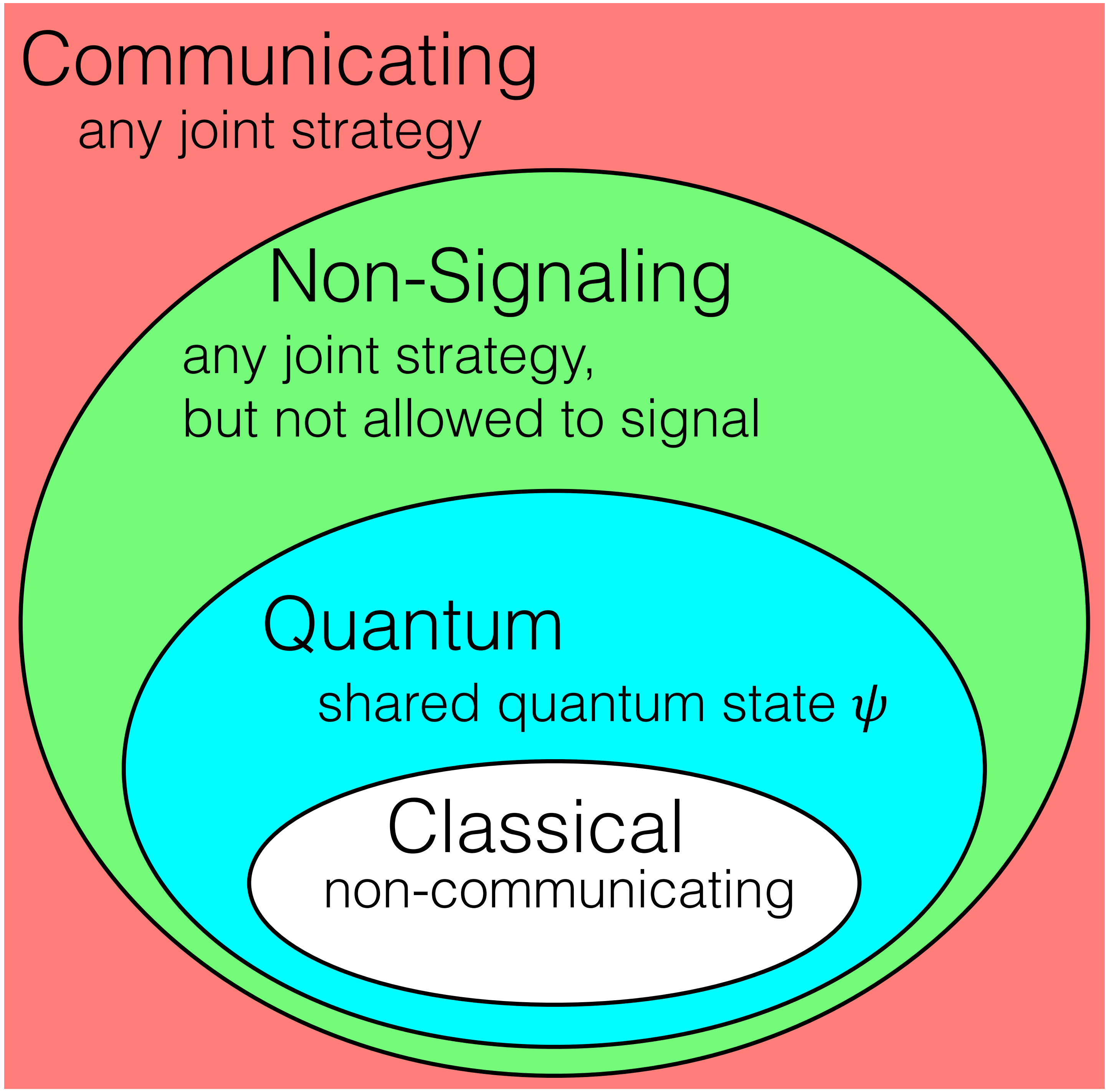

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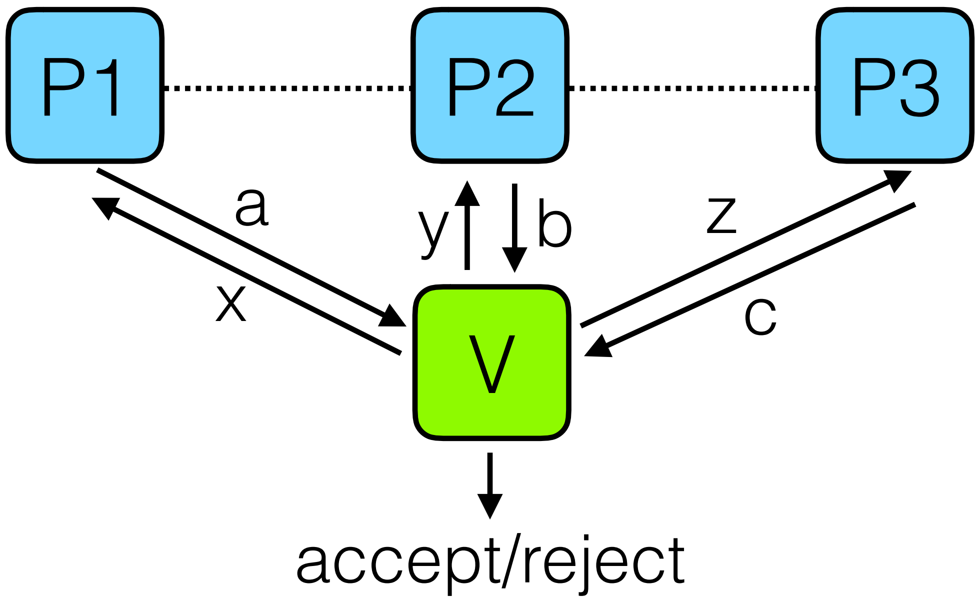

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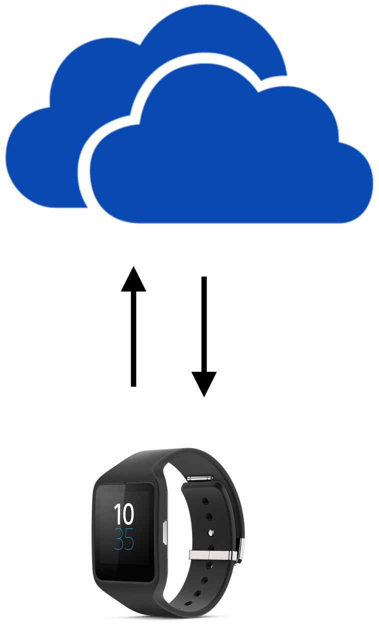

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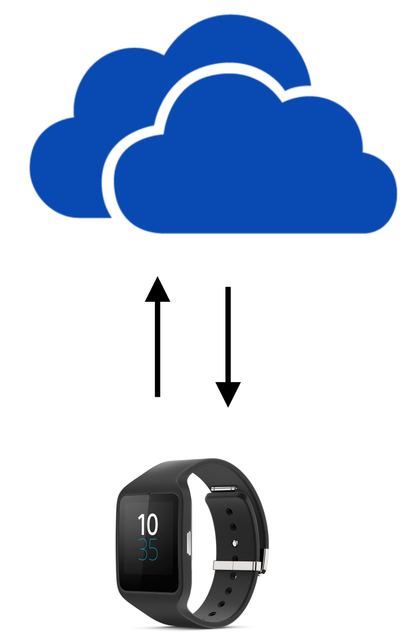

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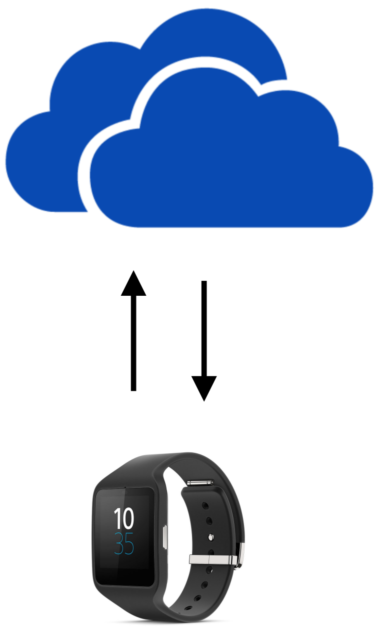

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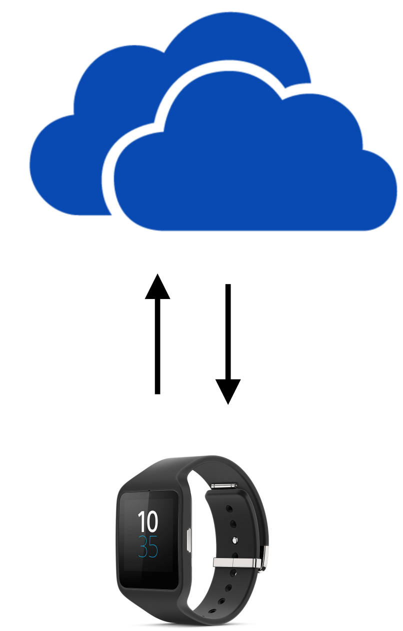

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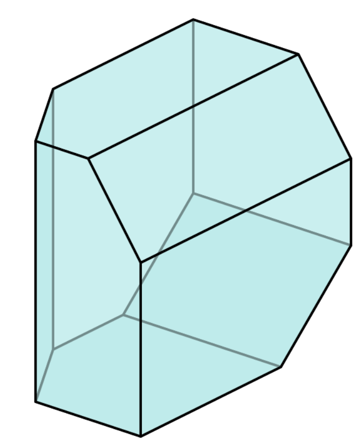
nsMIP —[KRR14]→ hardness of approximation for LPs in bounded space



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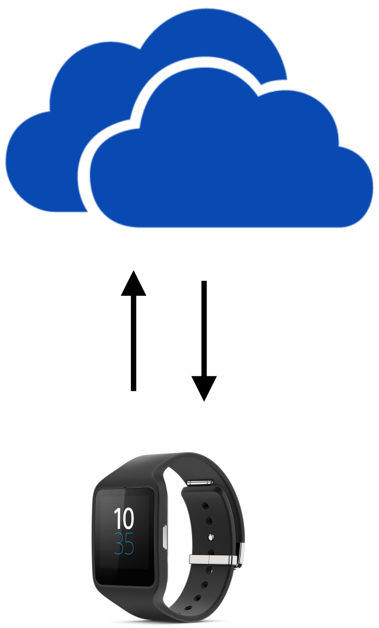

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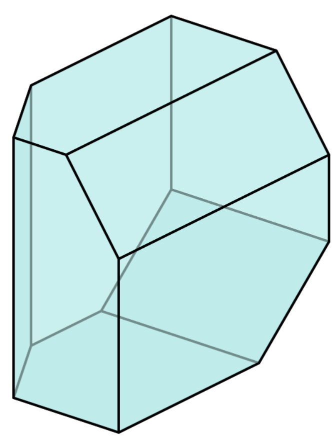

BUT: current nsMIP constructions appear *sub-optimal*

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Standard MIPs

PCP Theorem:
[AS98] [ALMSS98]

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Best-understood case in classical setting

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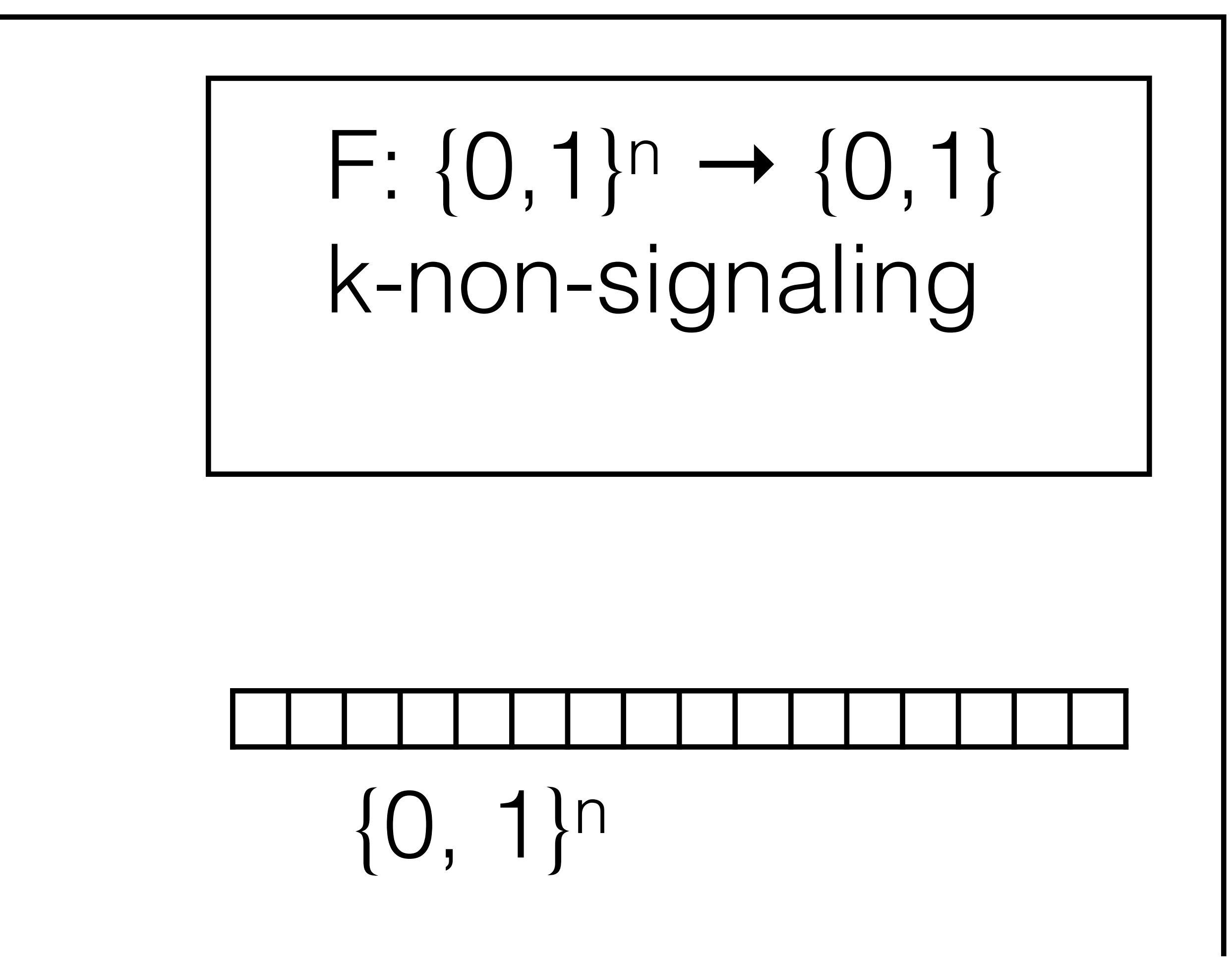

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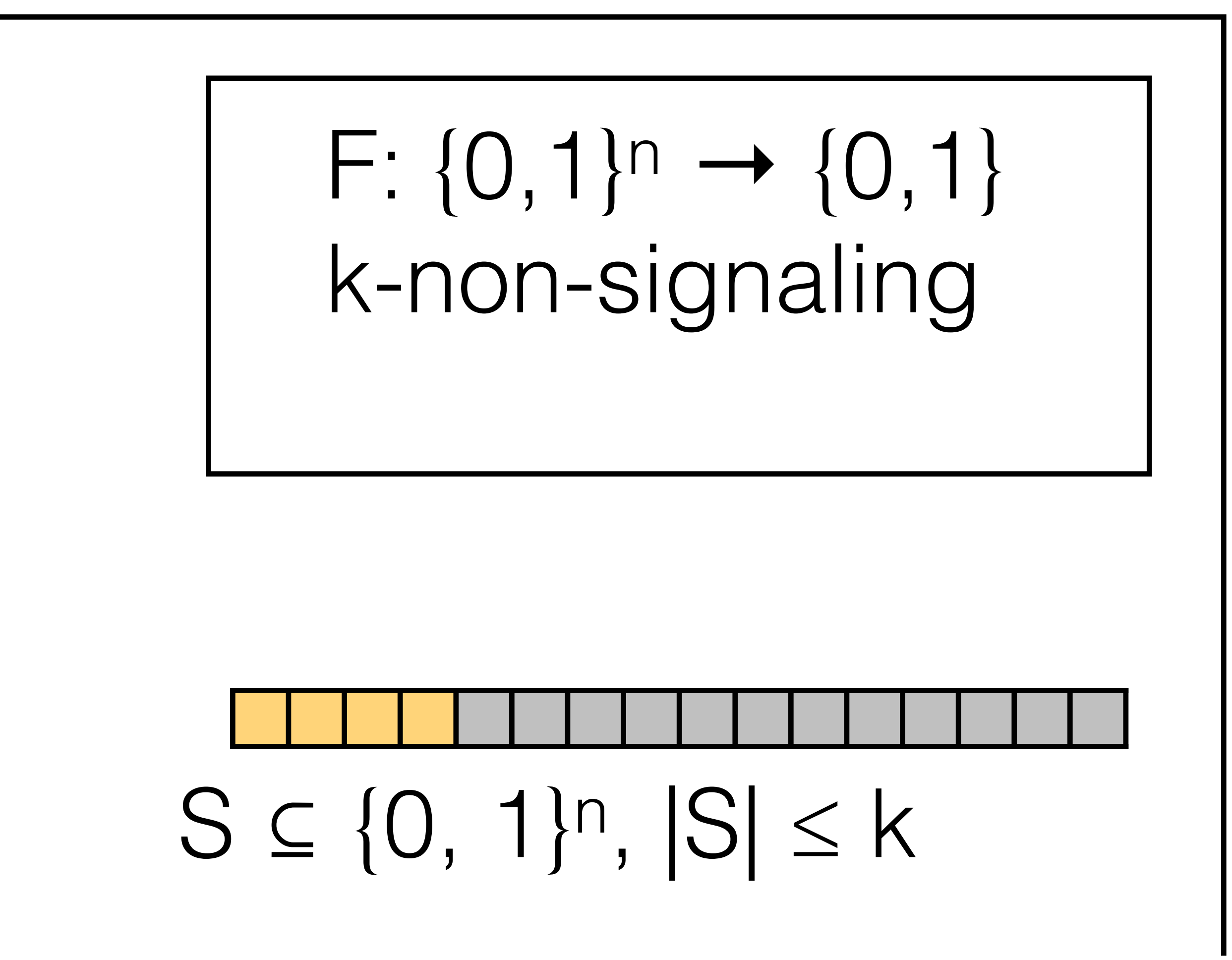

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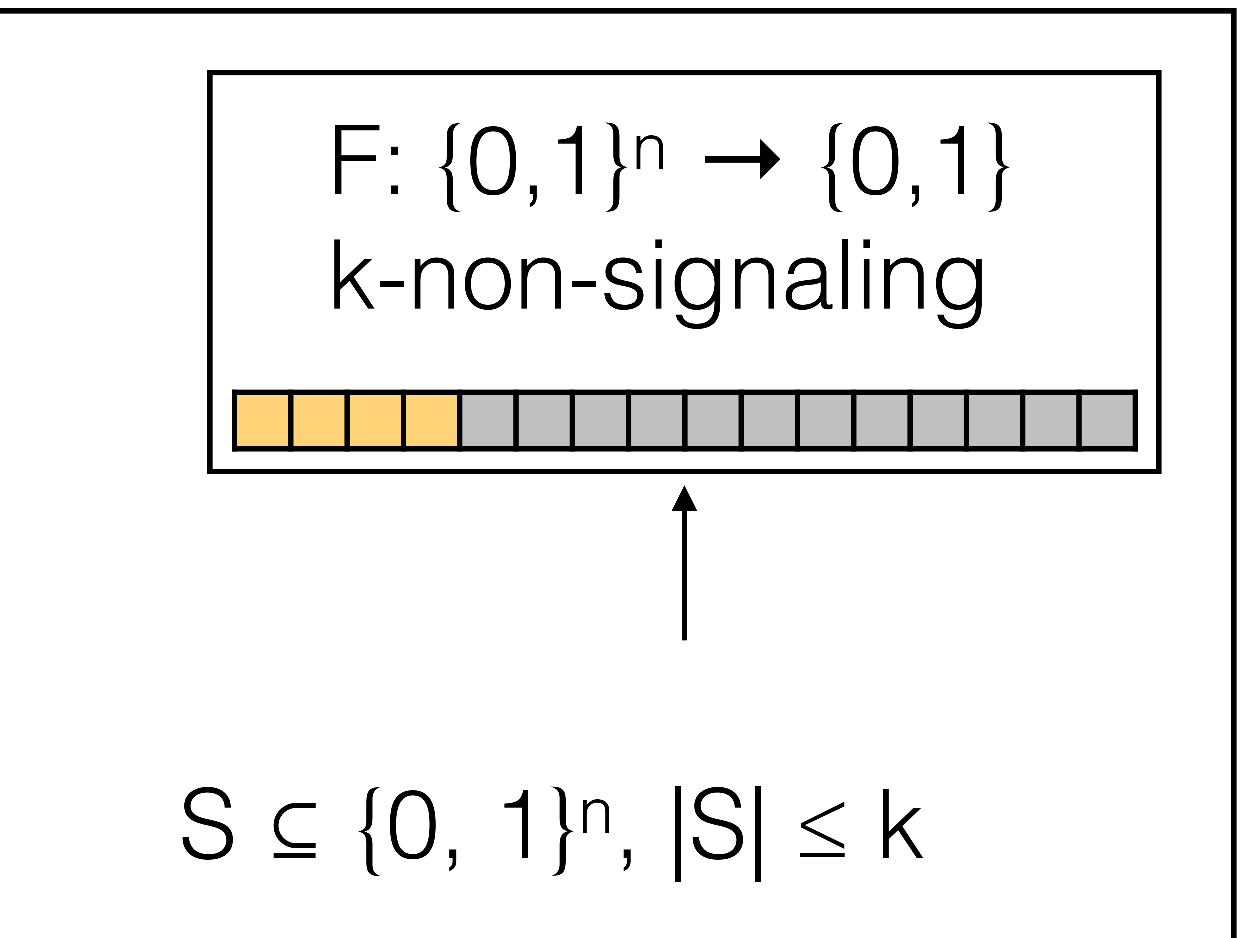

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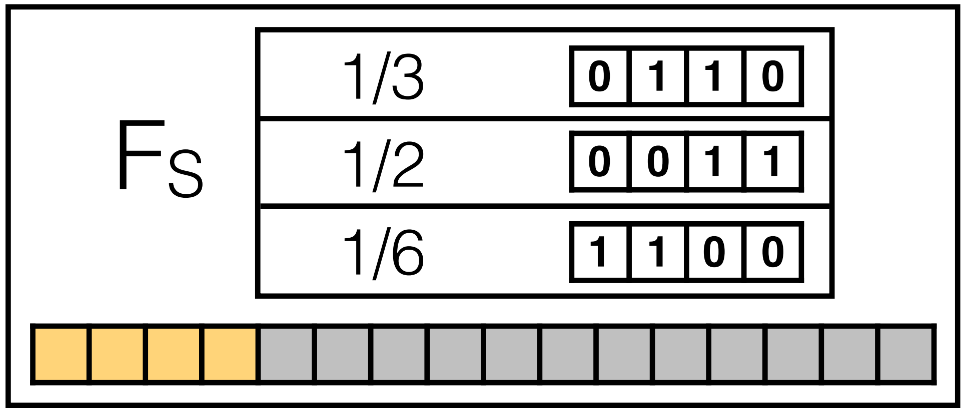

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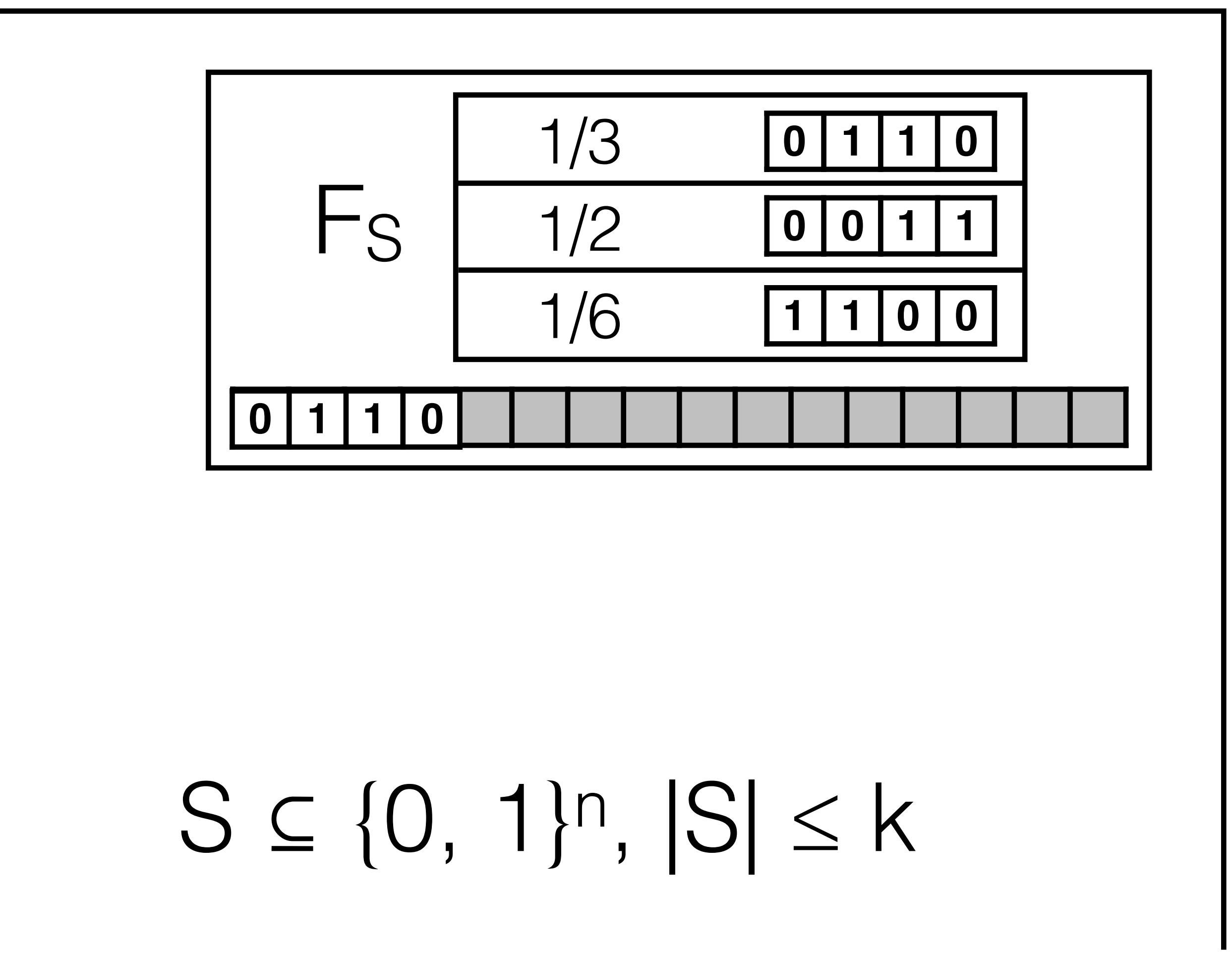

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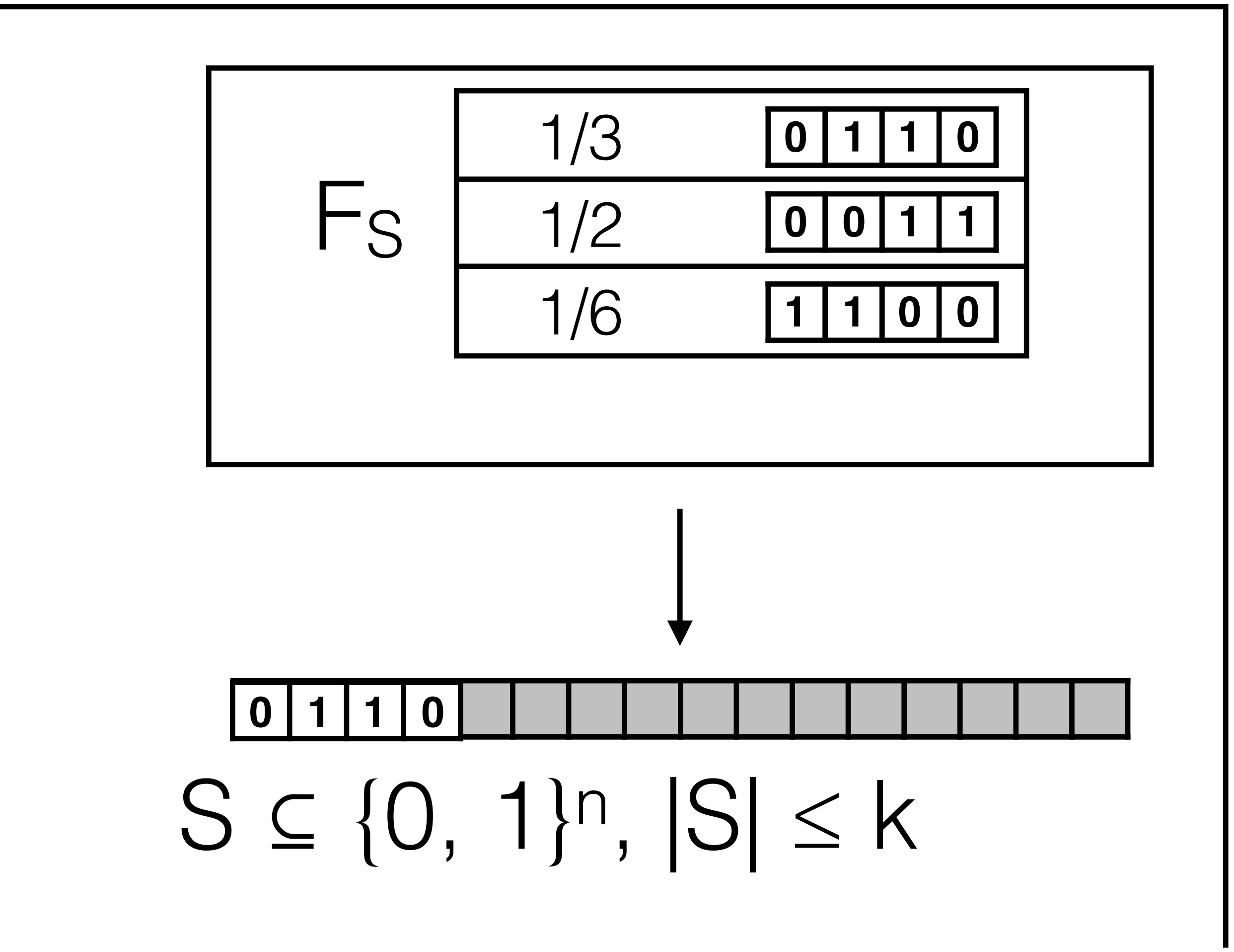

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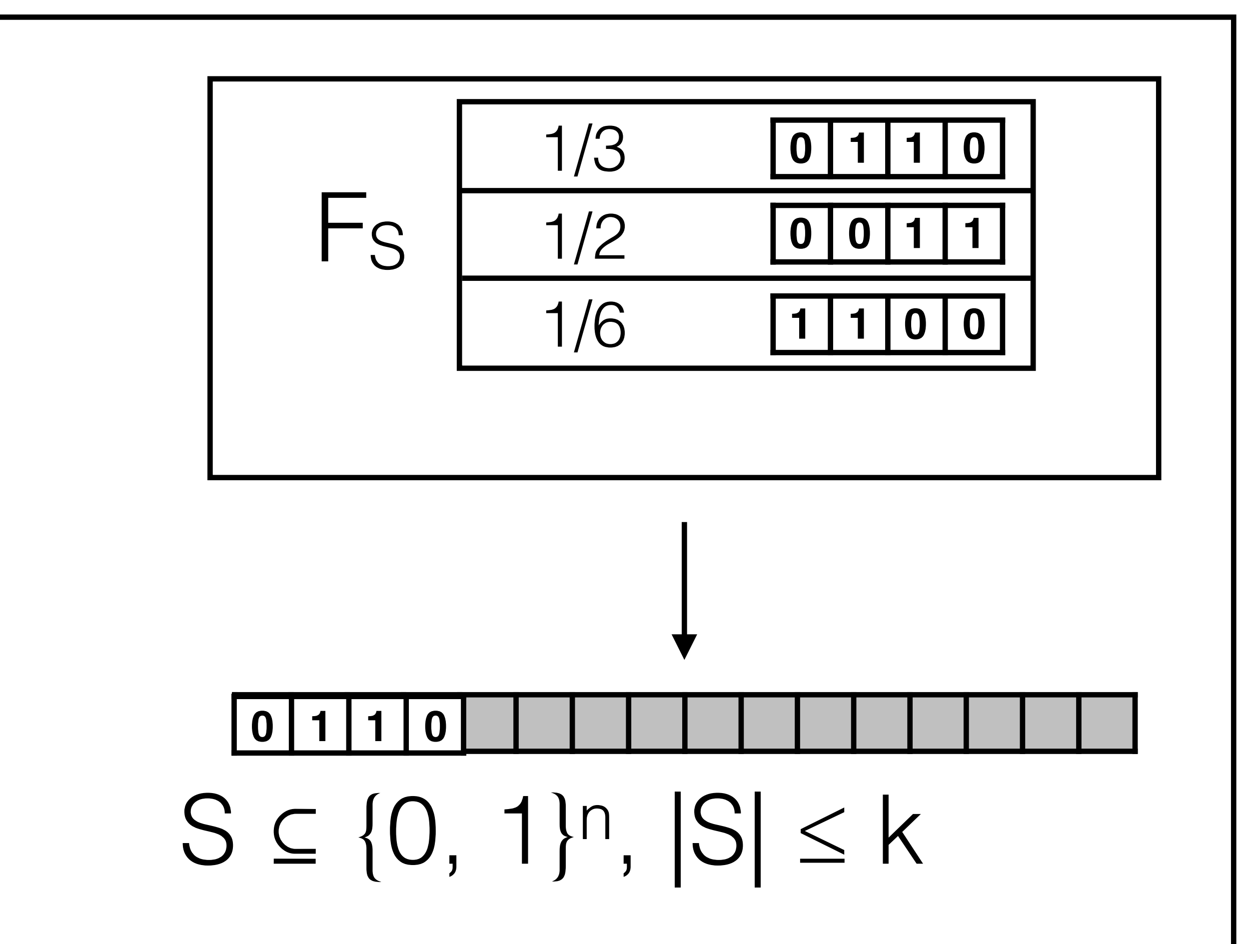
$$\forall S, T \subseteq \{0, 1\}^n, |S|, |T| \leq k$$

$$F_S|_{S \cap T} \equiv F_T|_{S \cap T}$$

(the marginal distributions are equal)



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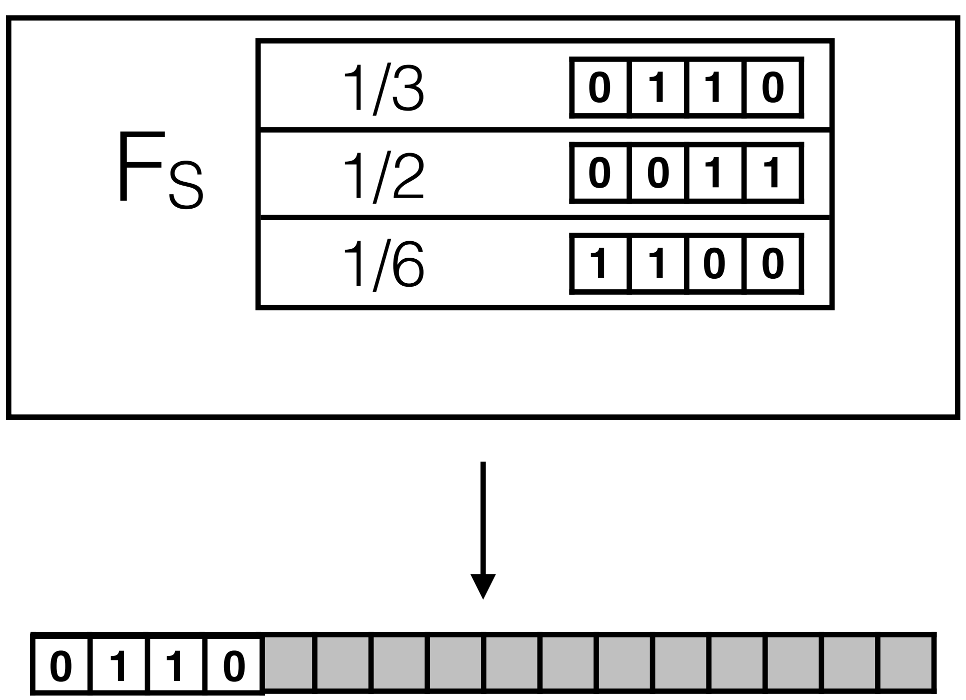

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F_S

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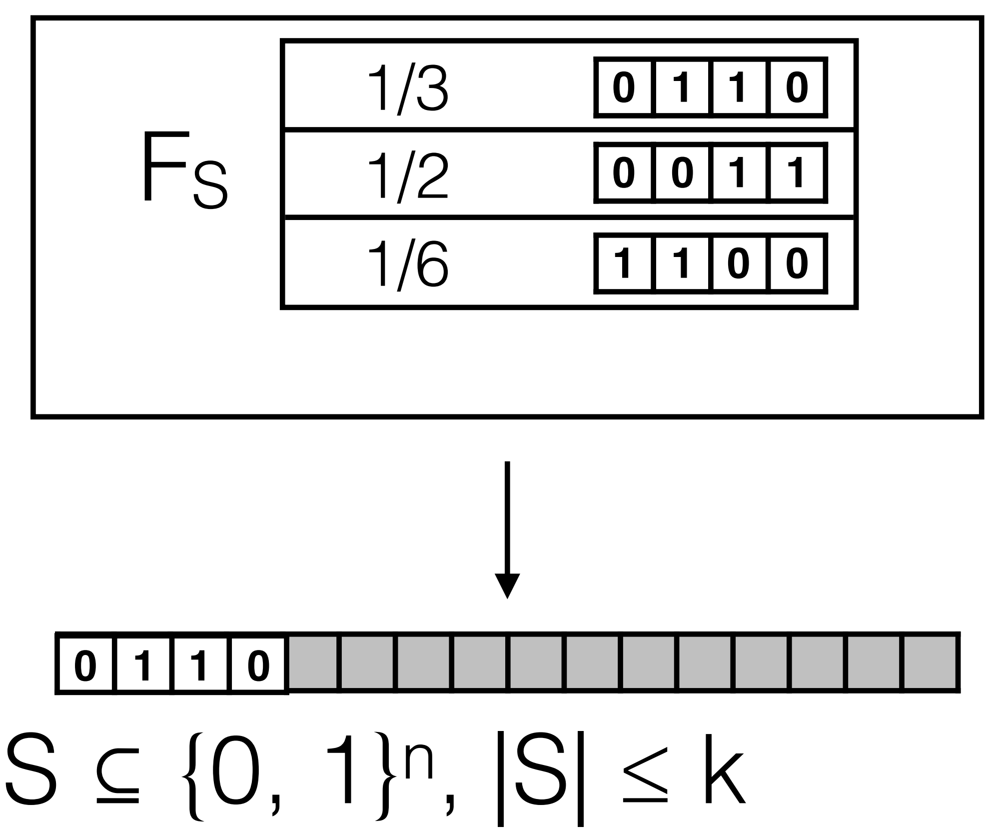

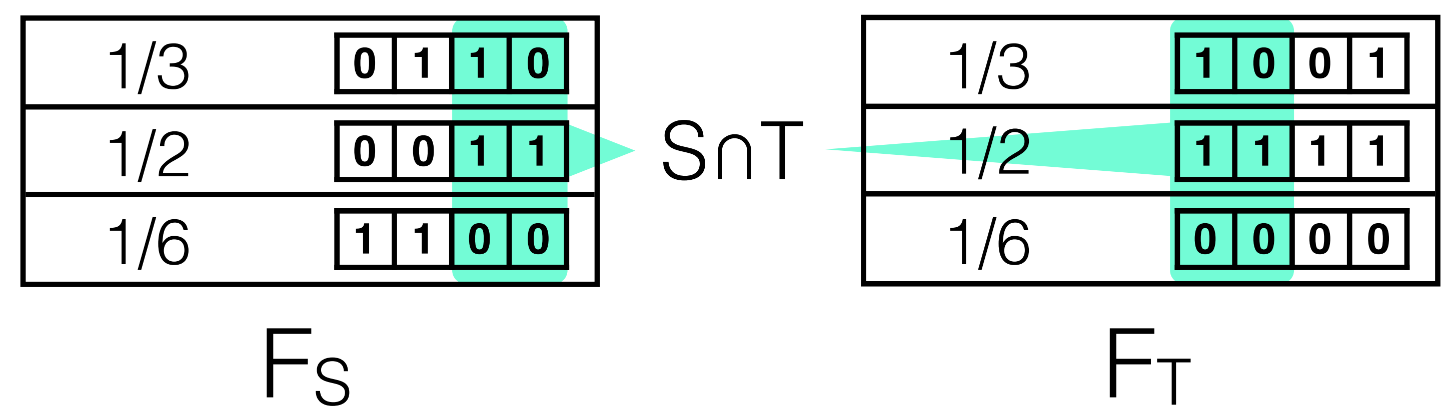

Non-Signaling Functions

Definition:

A k-non-signaling function $F: \{0,1\}^n \rightarrow \{0,1\}$ is a collection of distributions $\{F_S\}_S$ over functions $f: S \rightarrow \{0,1\}$, $\forall\, S \subseteq \{0, 1\}^n$, $|S| \leq k$ that satisfies the **non-signaling property:**

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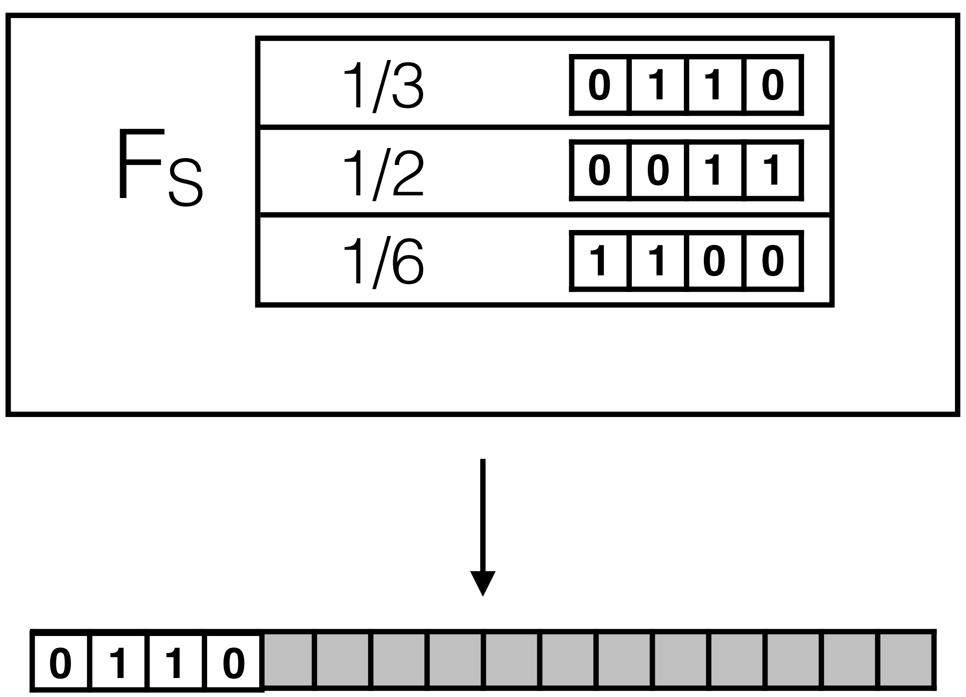

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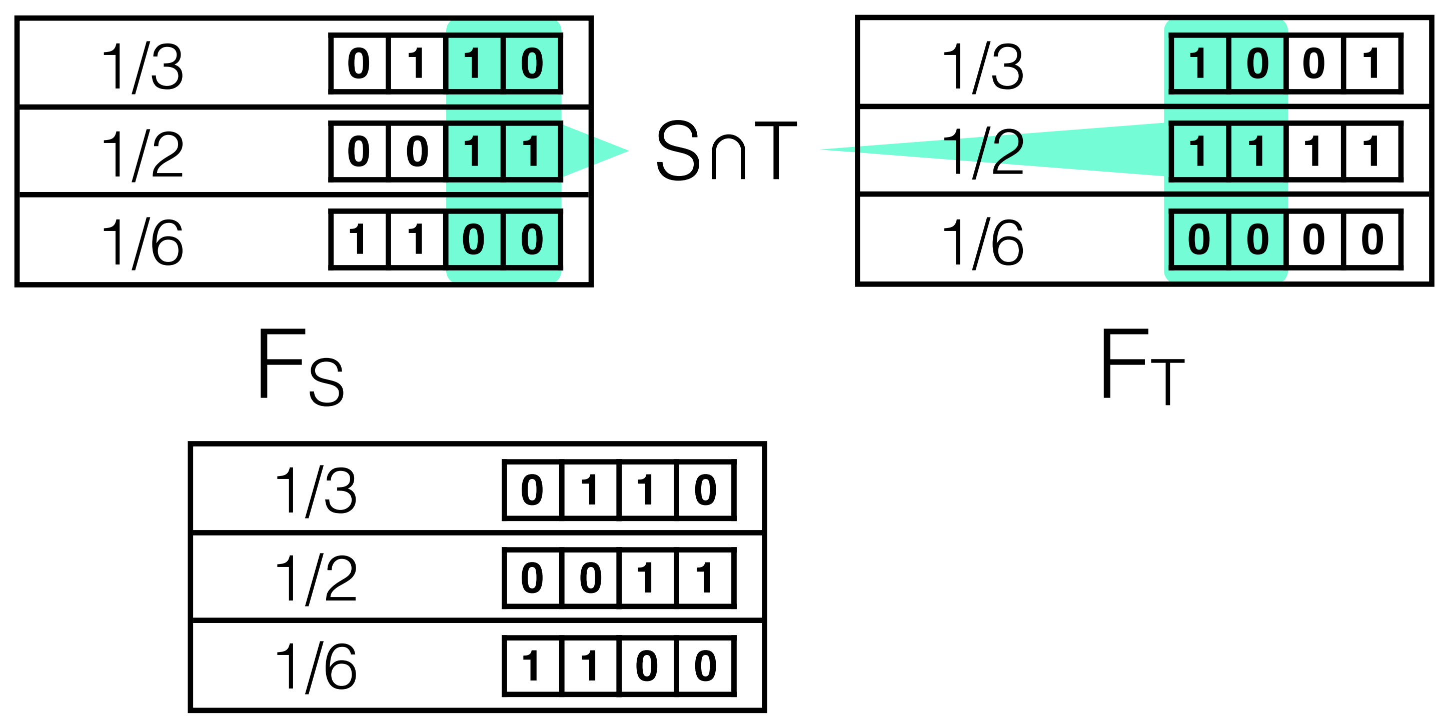

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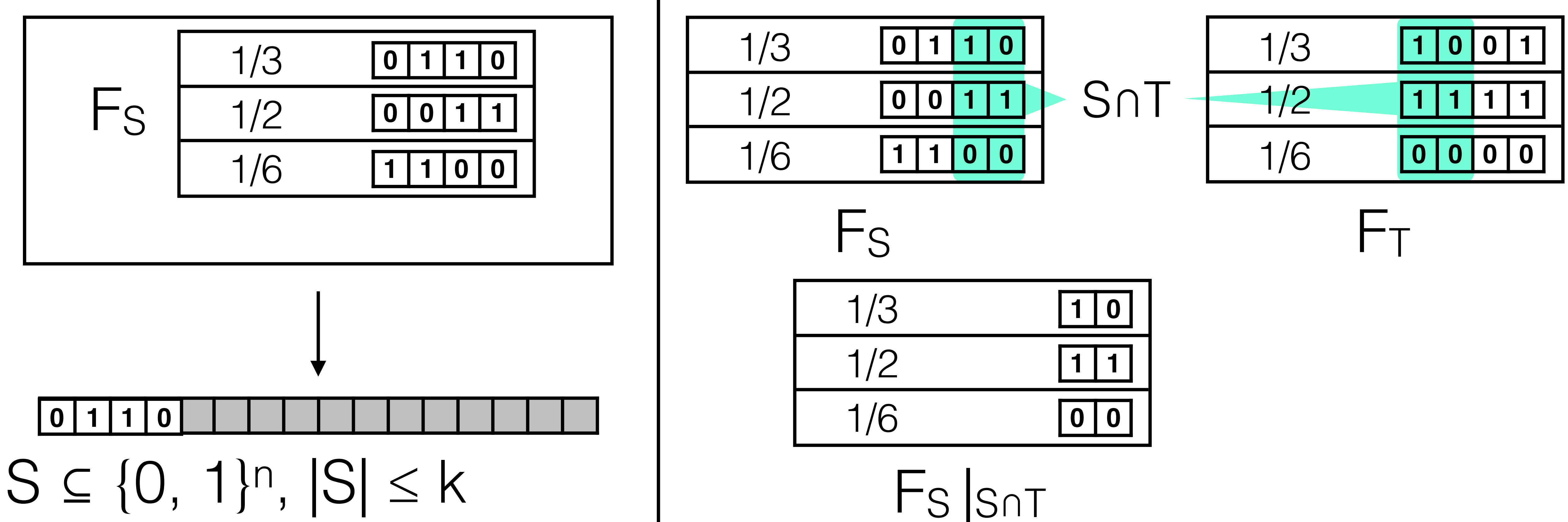

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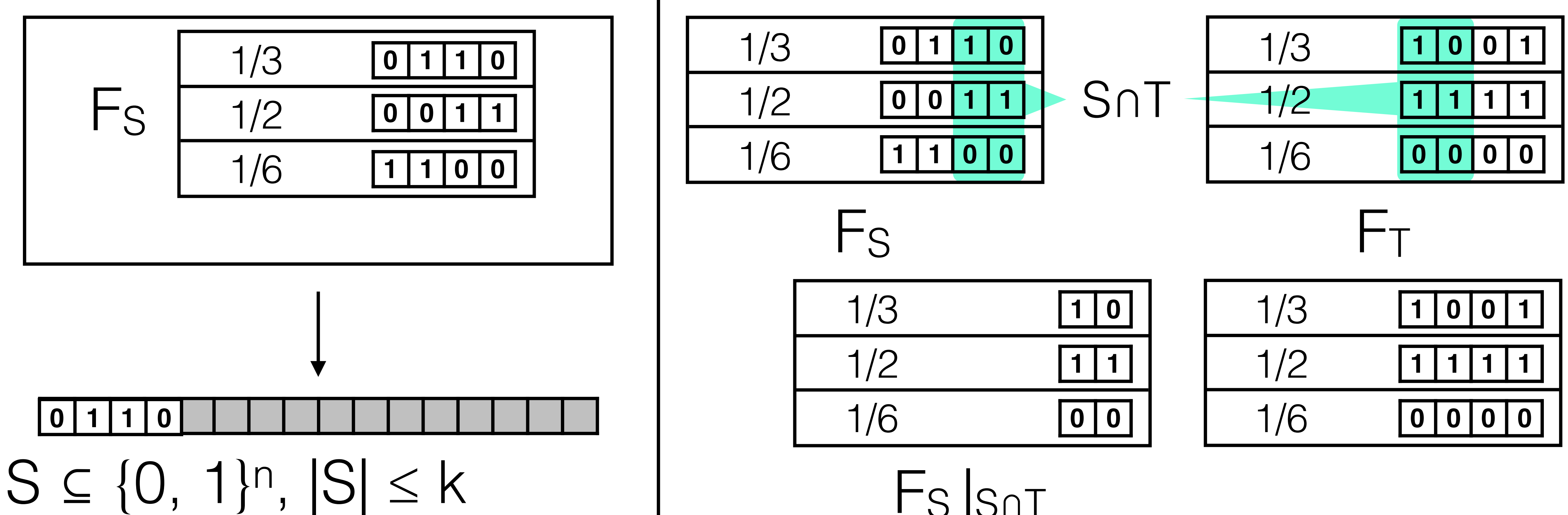

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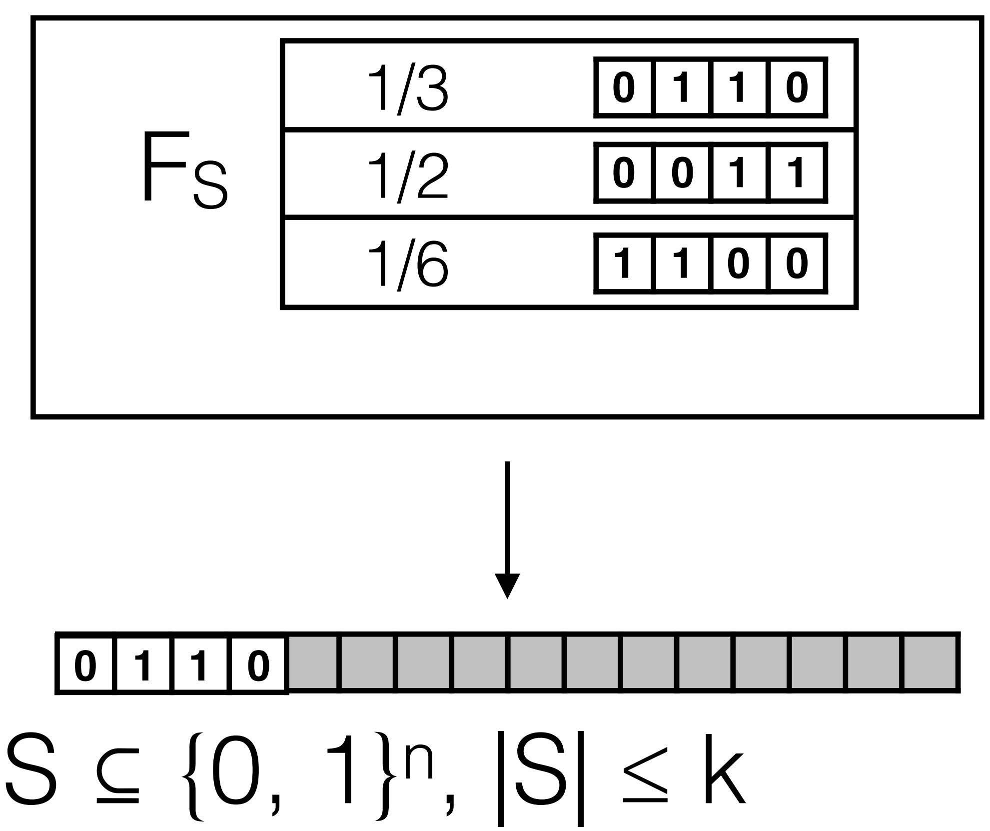

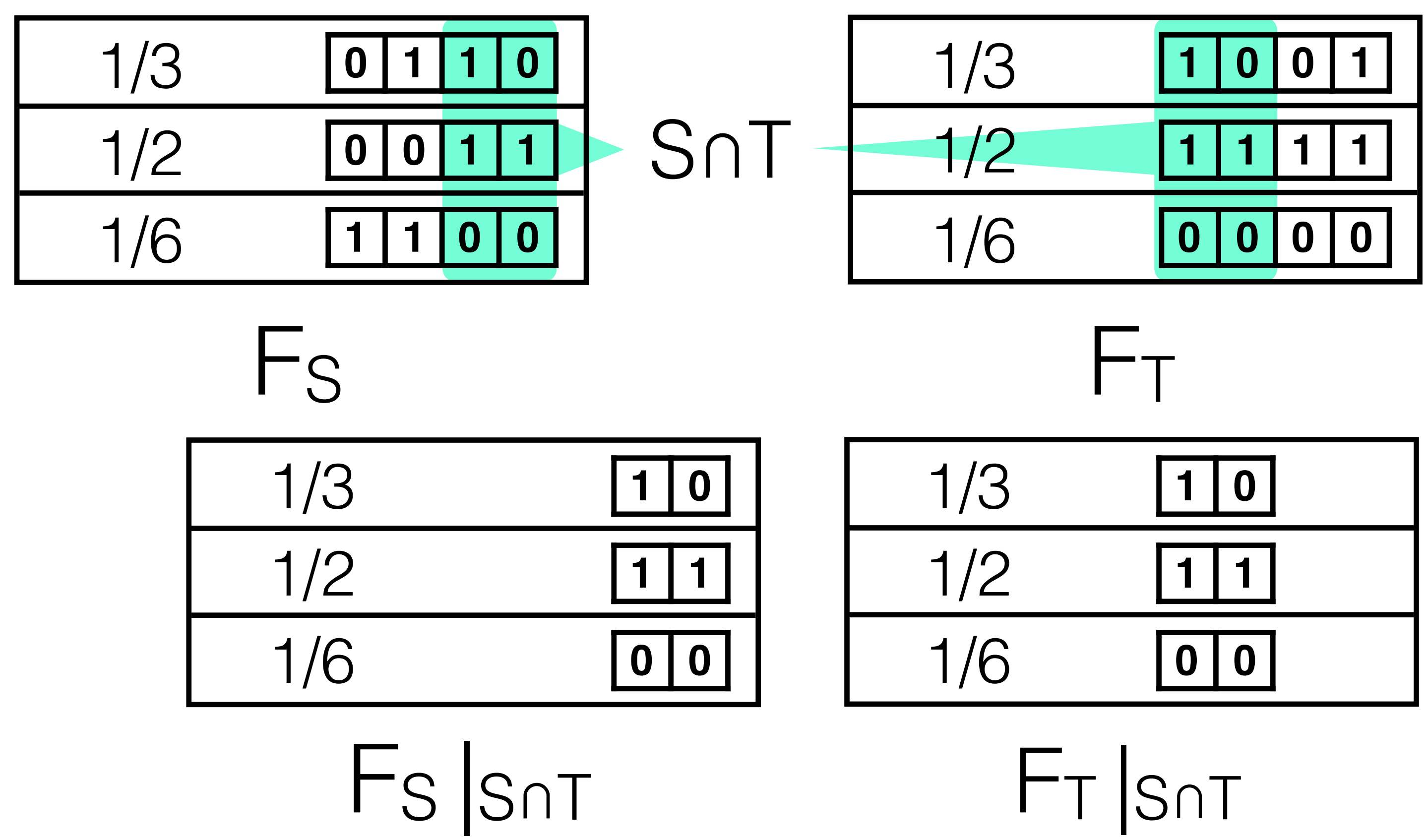

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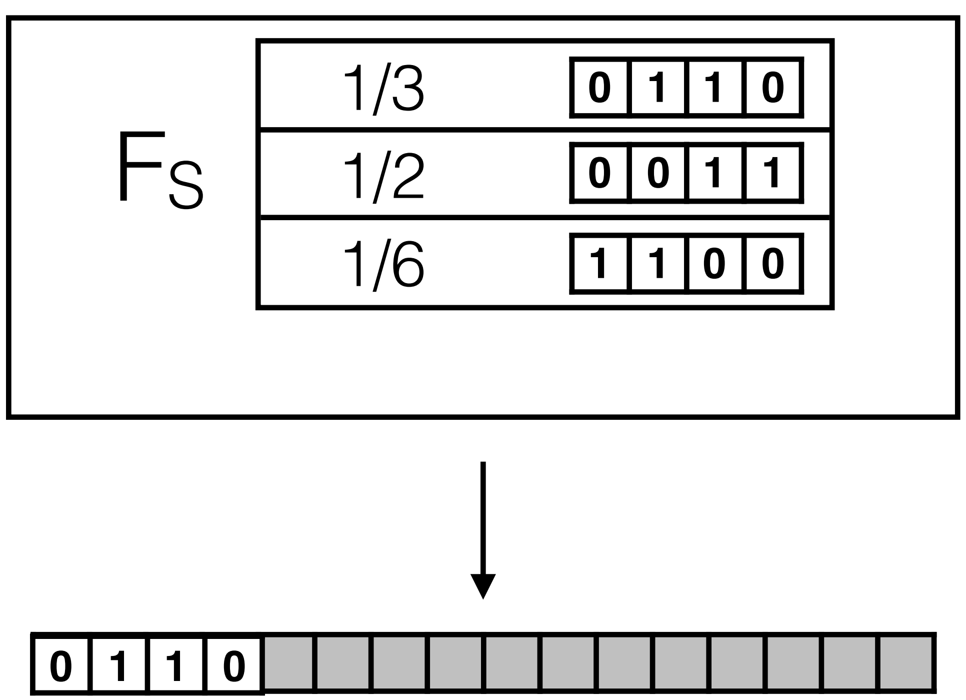

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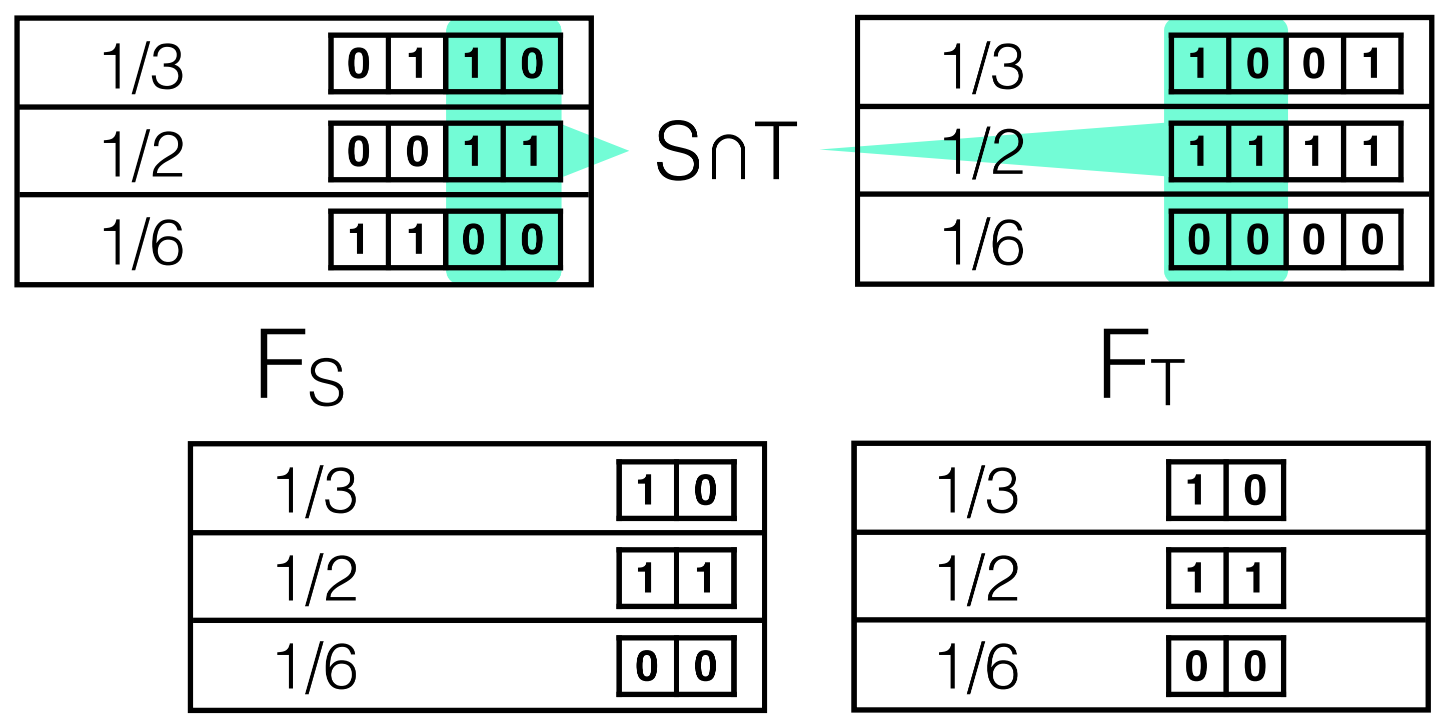

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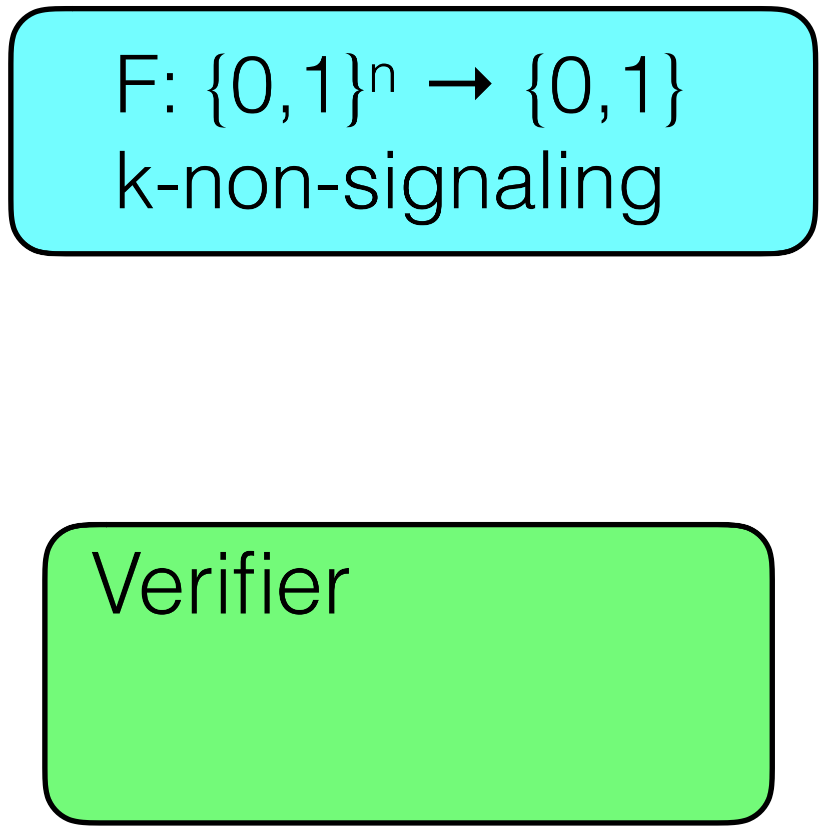

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k-non-signaling

Verifier
$x,y \leftarrow \{0,1\}^n$

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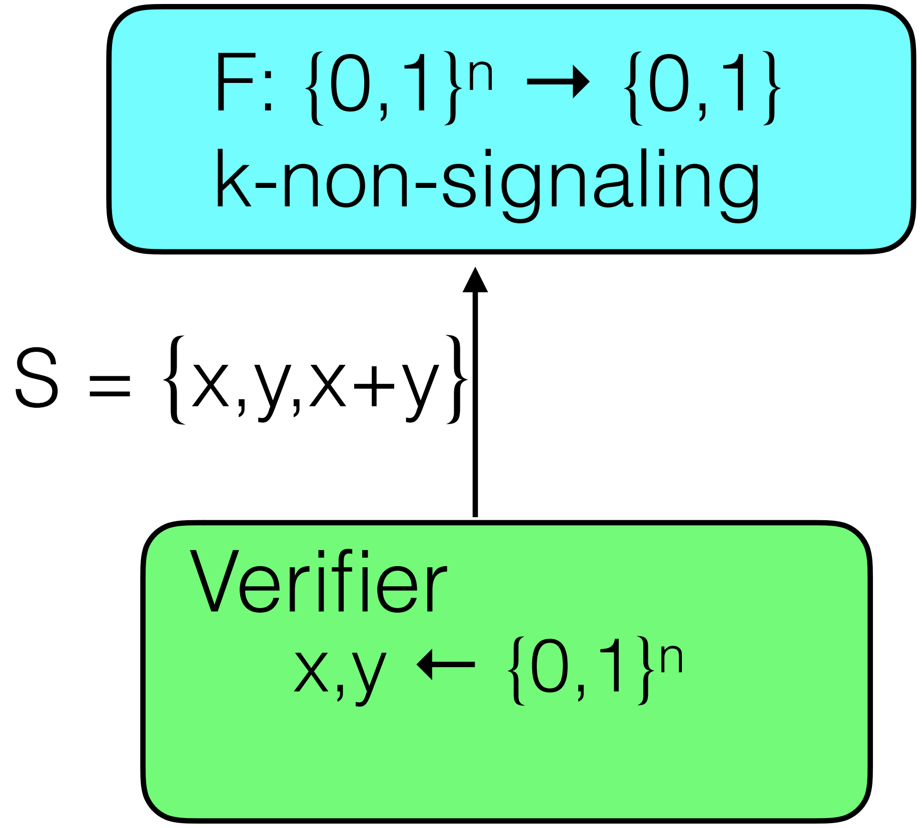

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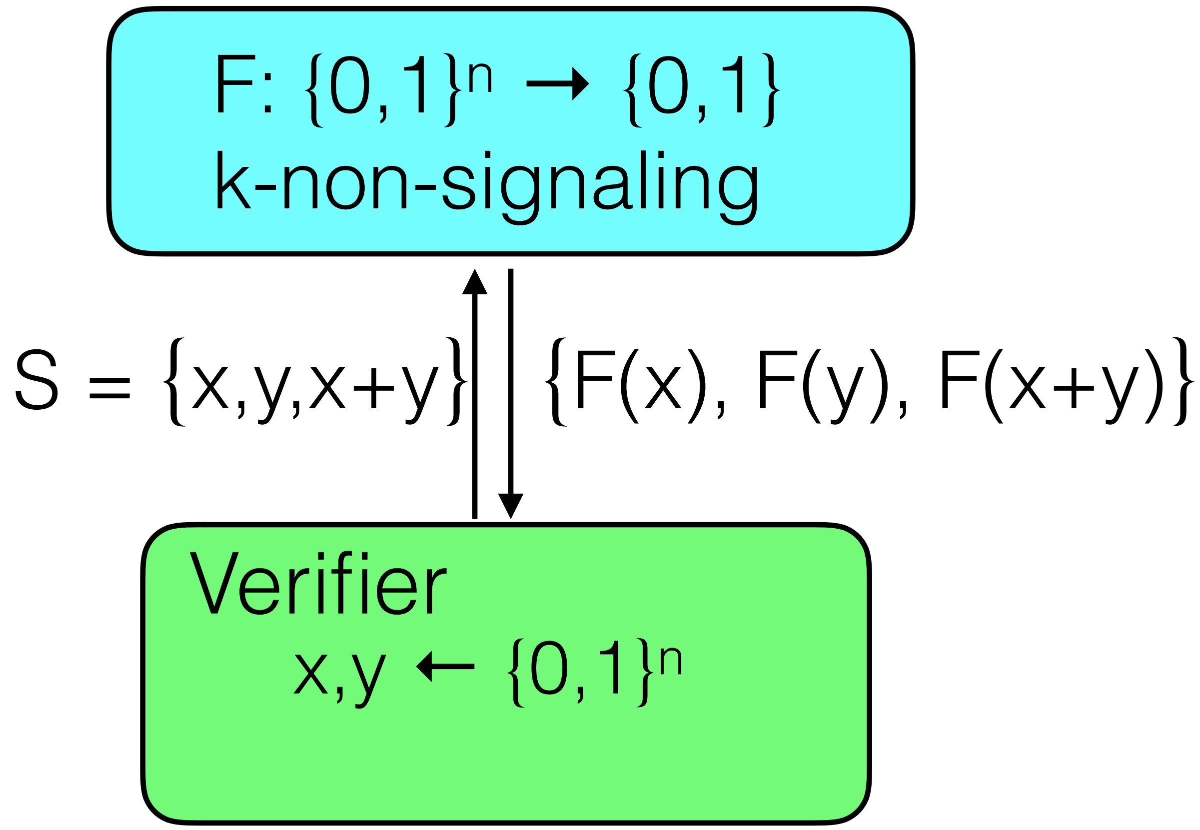

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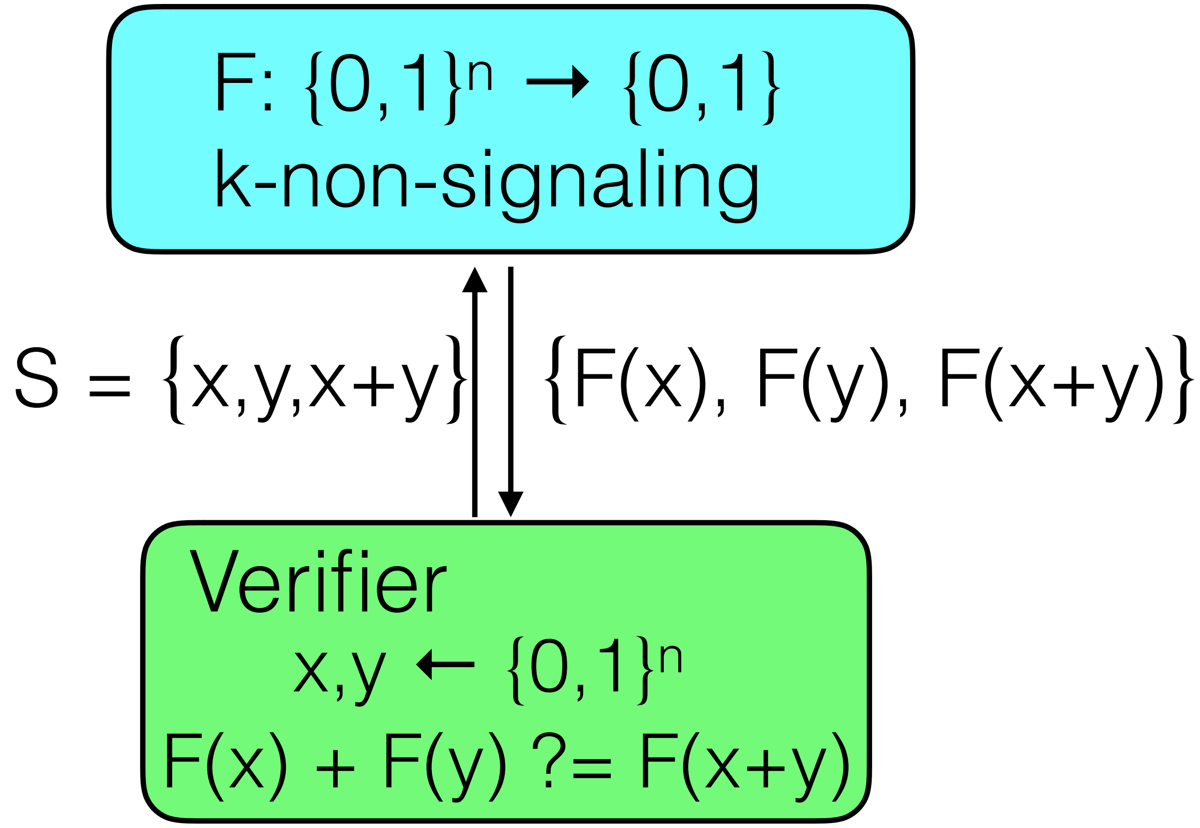

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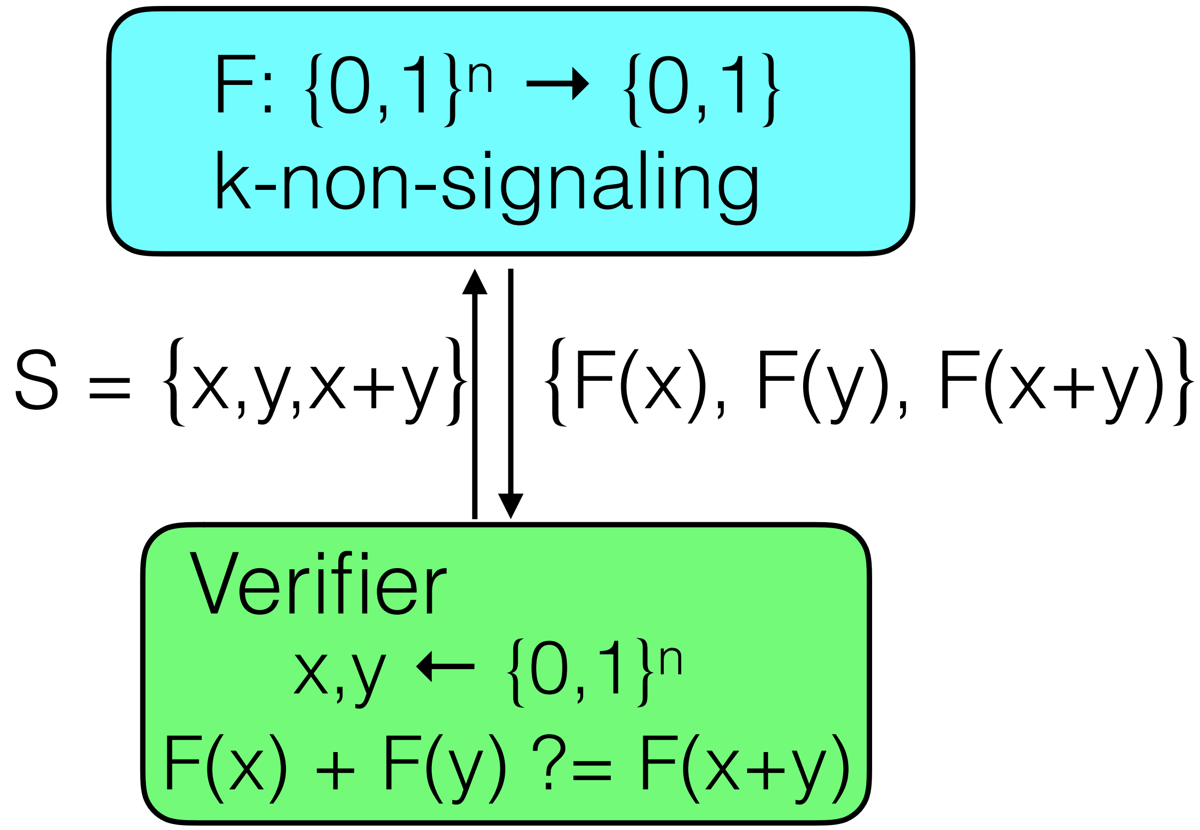

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$\Pr_{x,y,F}[F \text{ passes}] = 1 \rightarrow$ some global conclusion?

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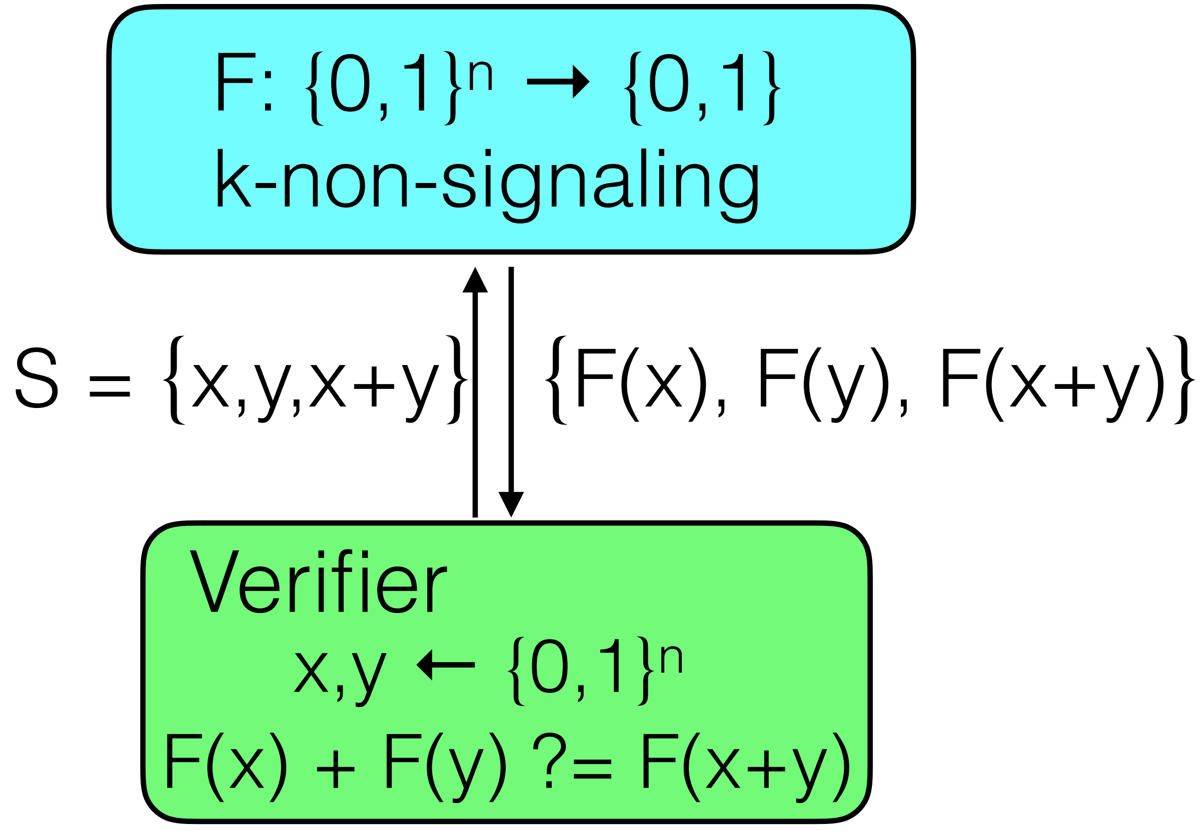

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How? NS functions are collections of local distributions.

Let's first understand non-signaling functions

Examples of NS Functions

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But can try anyways!

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Fact: system of linear equations has a solution.

Solution has *negative* entries, but marginals on "queryable sets" are non-negative.

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k-local quasi-distributions → k-non-signaling functions

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Does the reverse direction hold?

Theorem 1

k-local quasi-distributions ⟷ k-non-signaling functions

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Proof sketch:

⟹ direction: easy

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Theorem 1

k-local quasi-distributions $\Longleftrightarrow$ k-non-signaling functions

Proof sketch:

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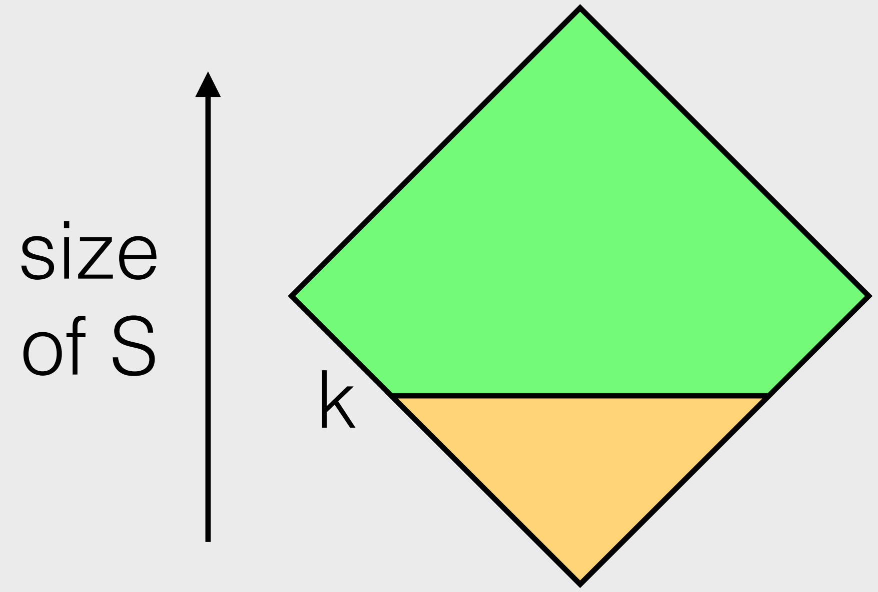

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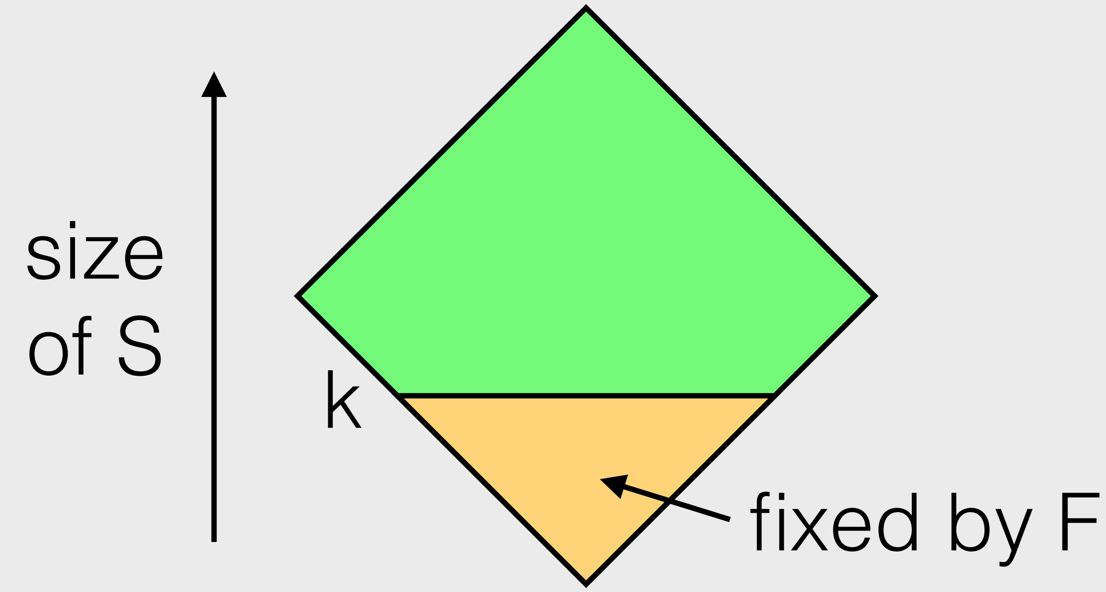

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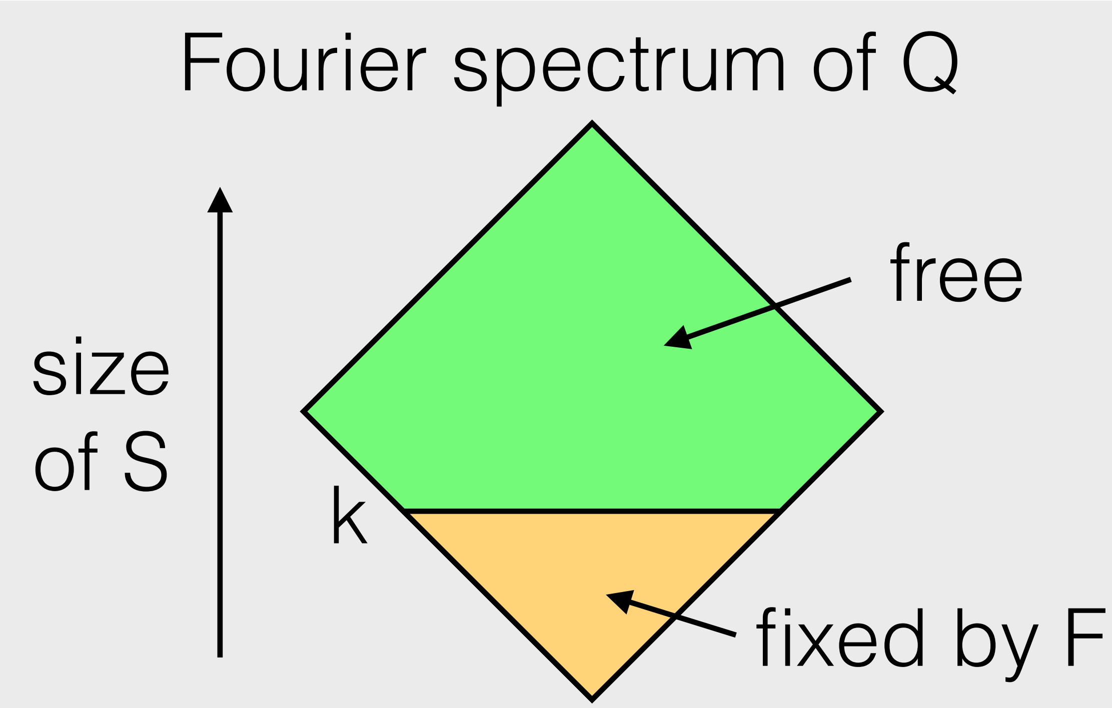

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F: $\{0,1\}^n$ -> $\{0,1\}$
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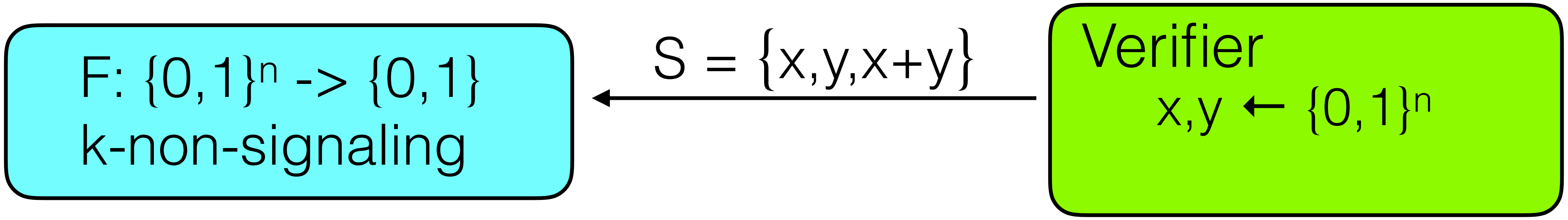

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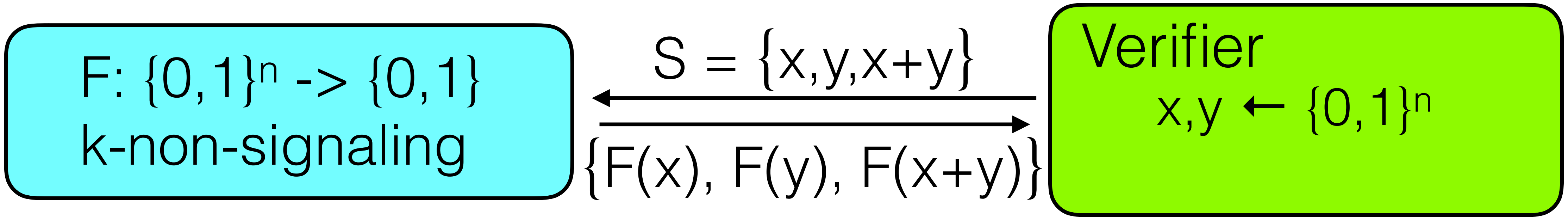

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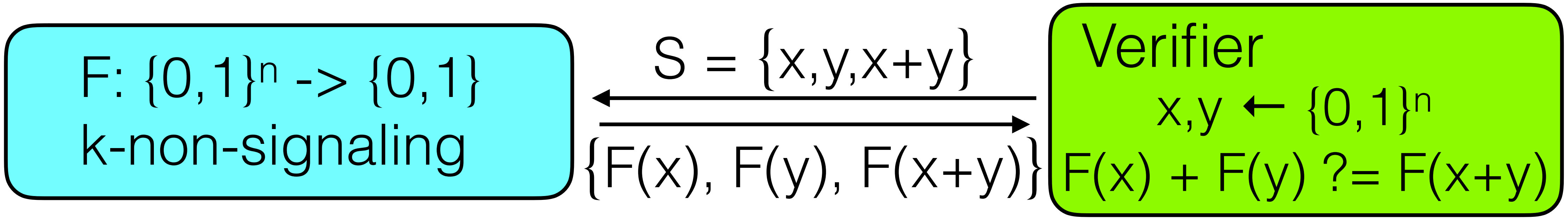

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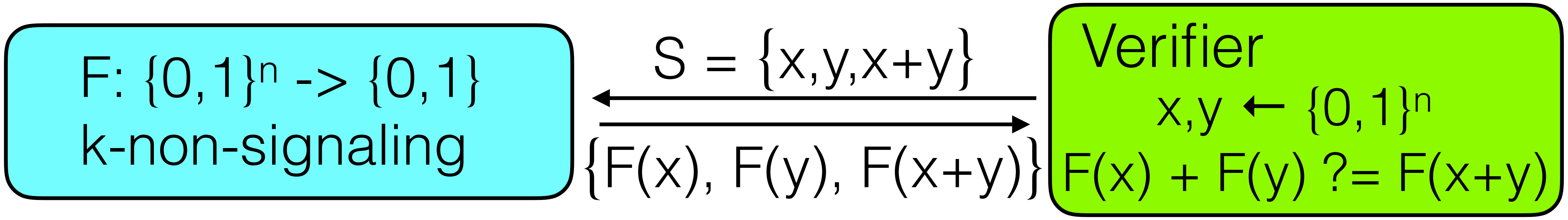

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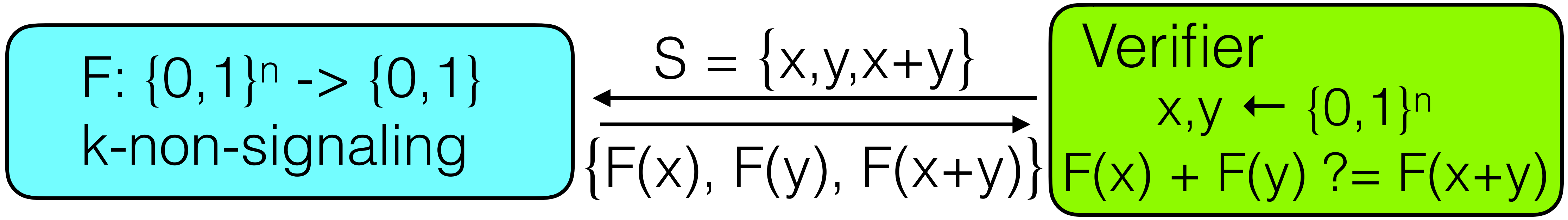

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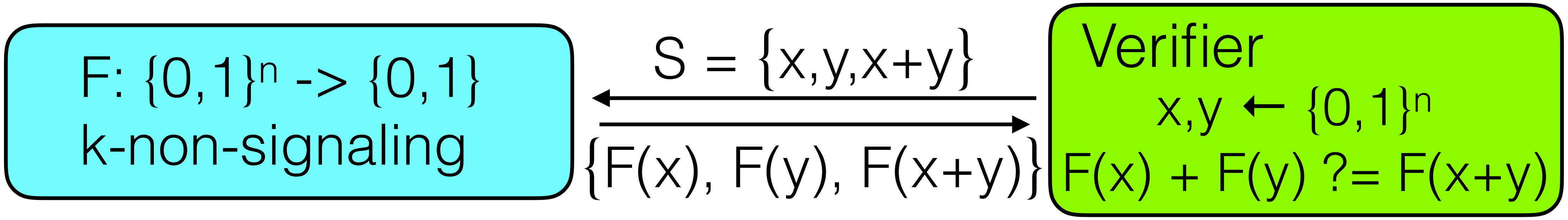

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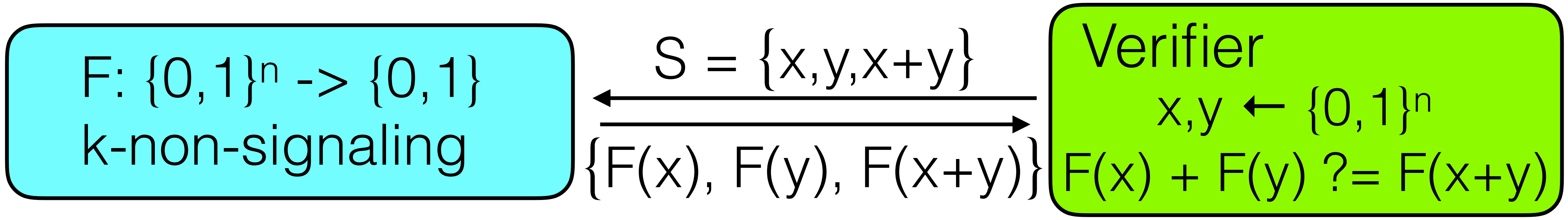

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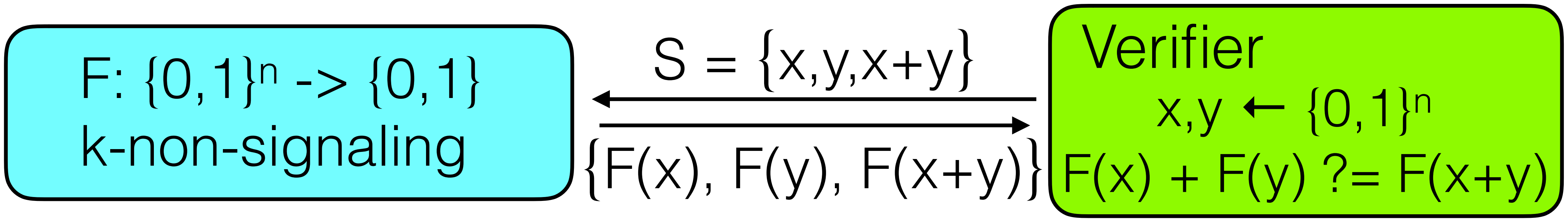

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This is a global conclusion!

Now suppose F passes the linearity test w.p. 1 - ε

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This is the correct answer.

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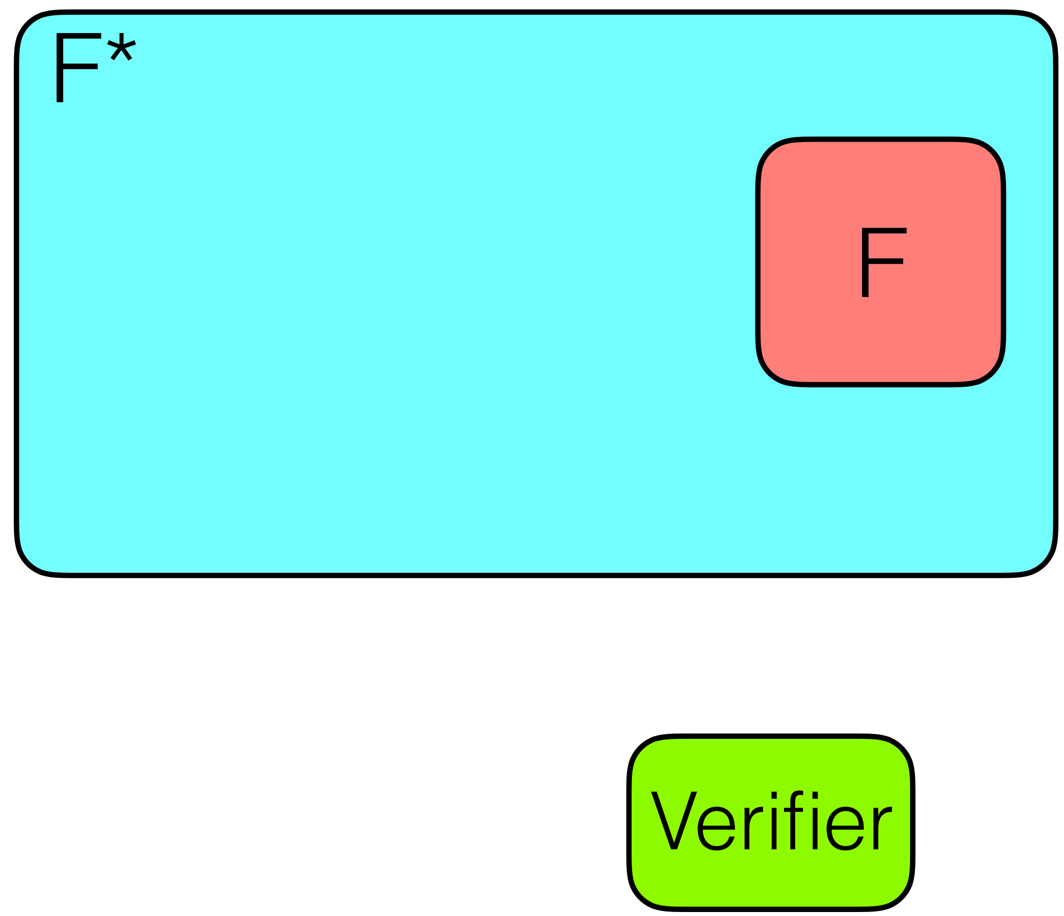

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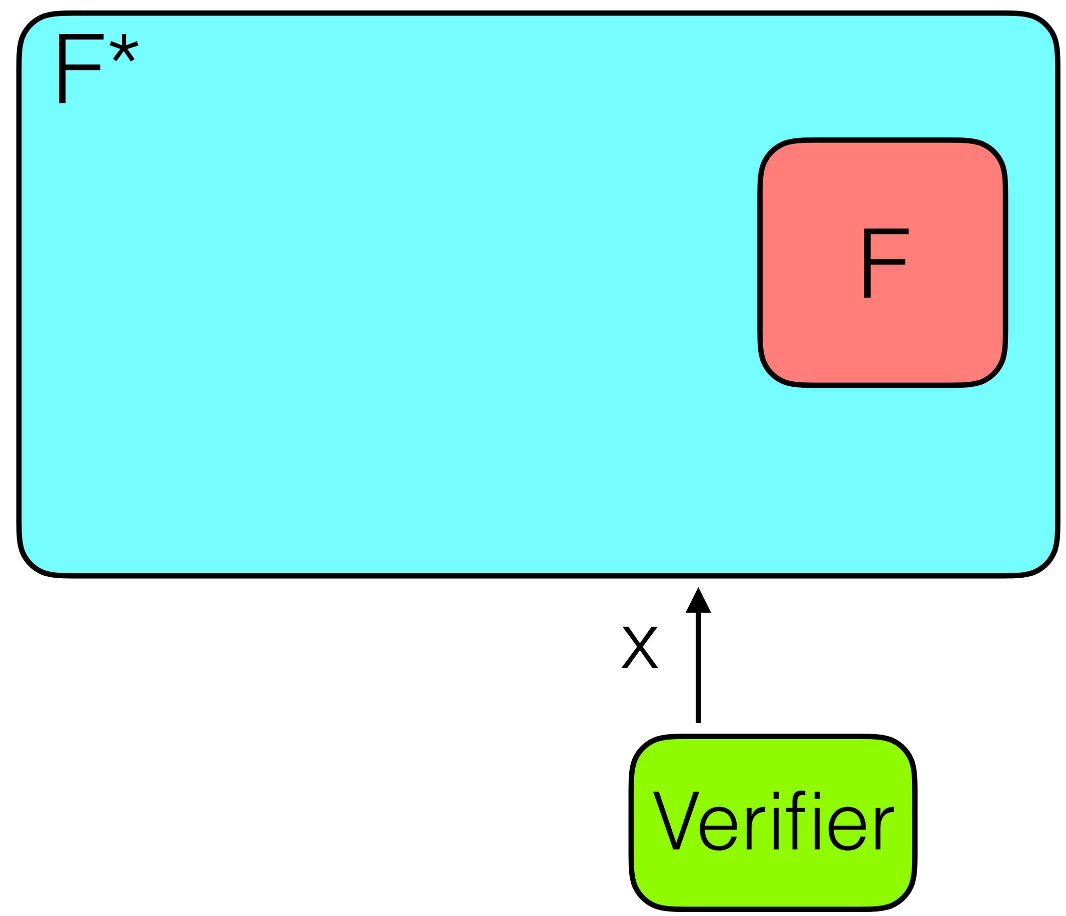

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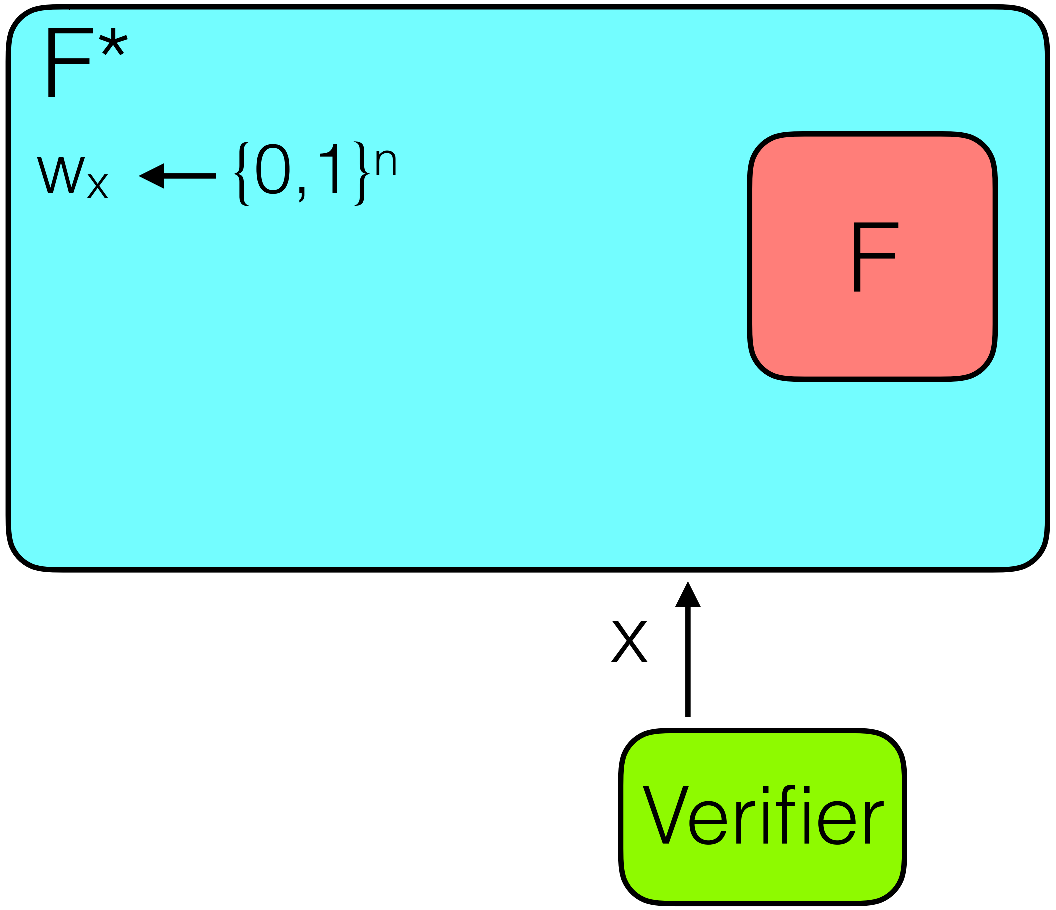

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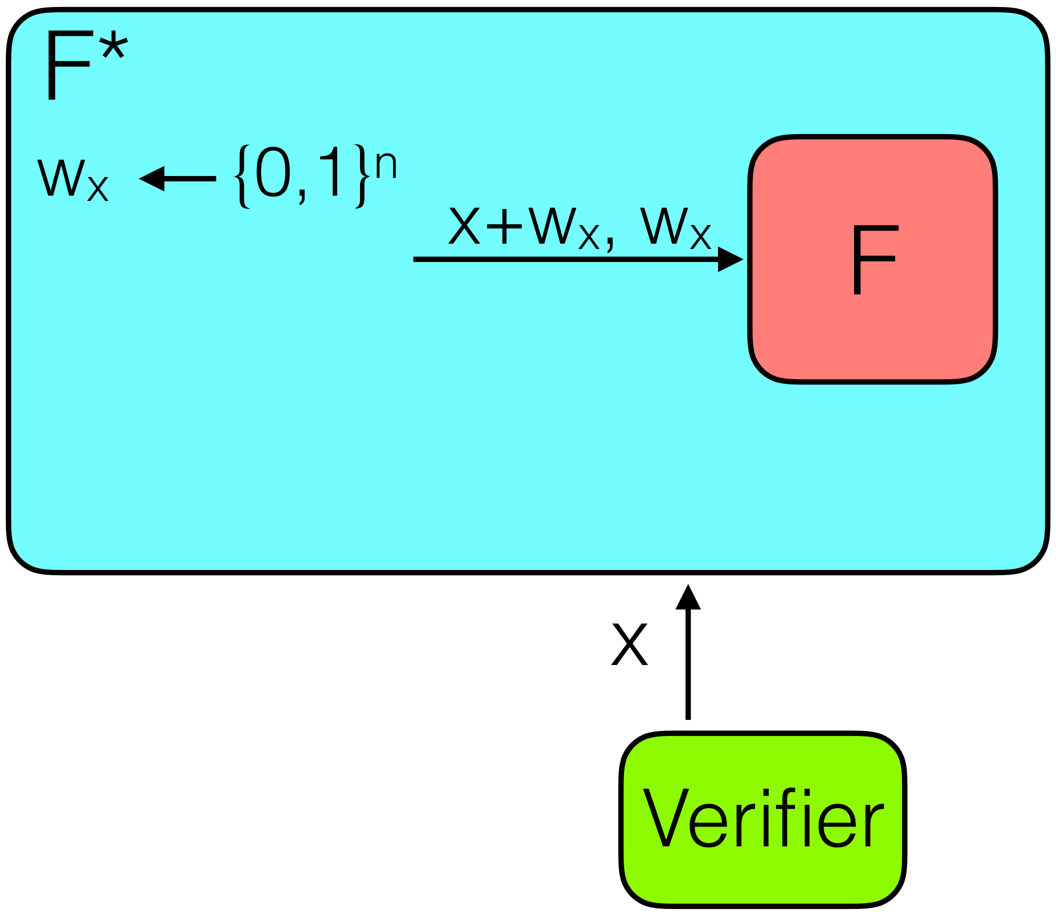

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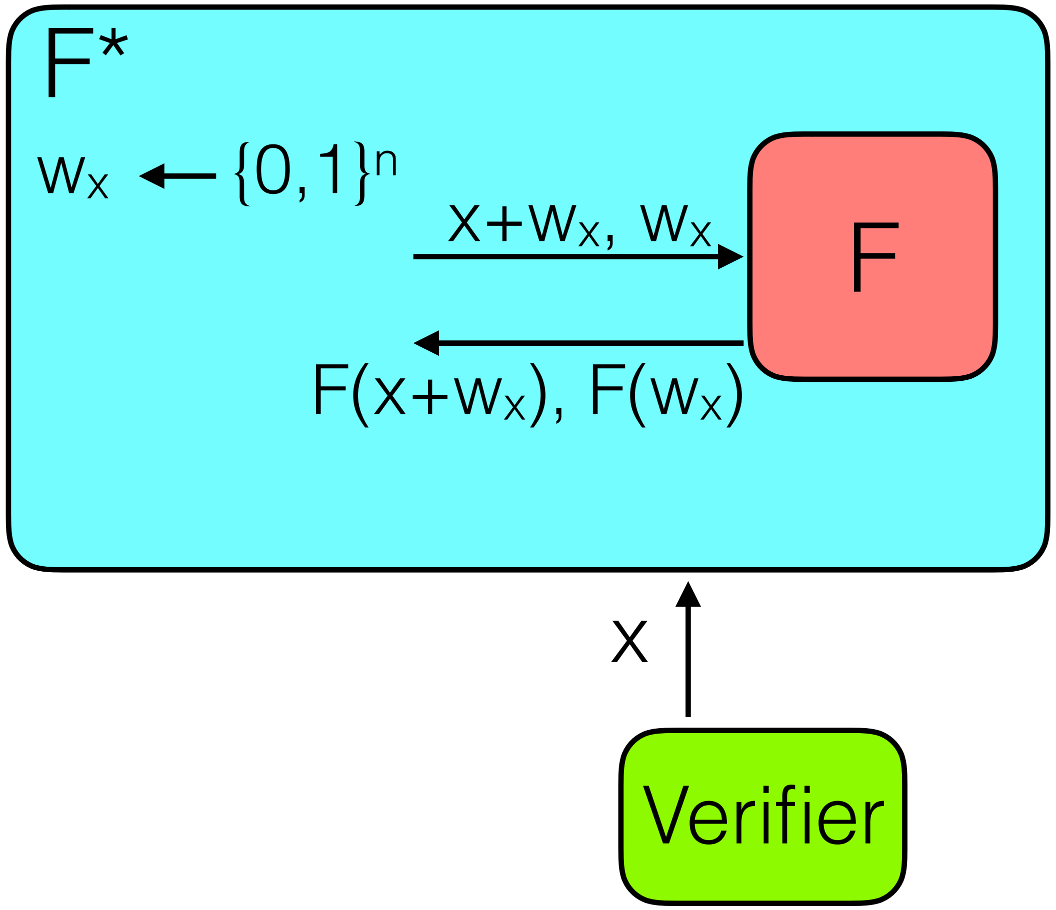

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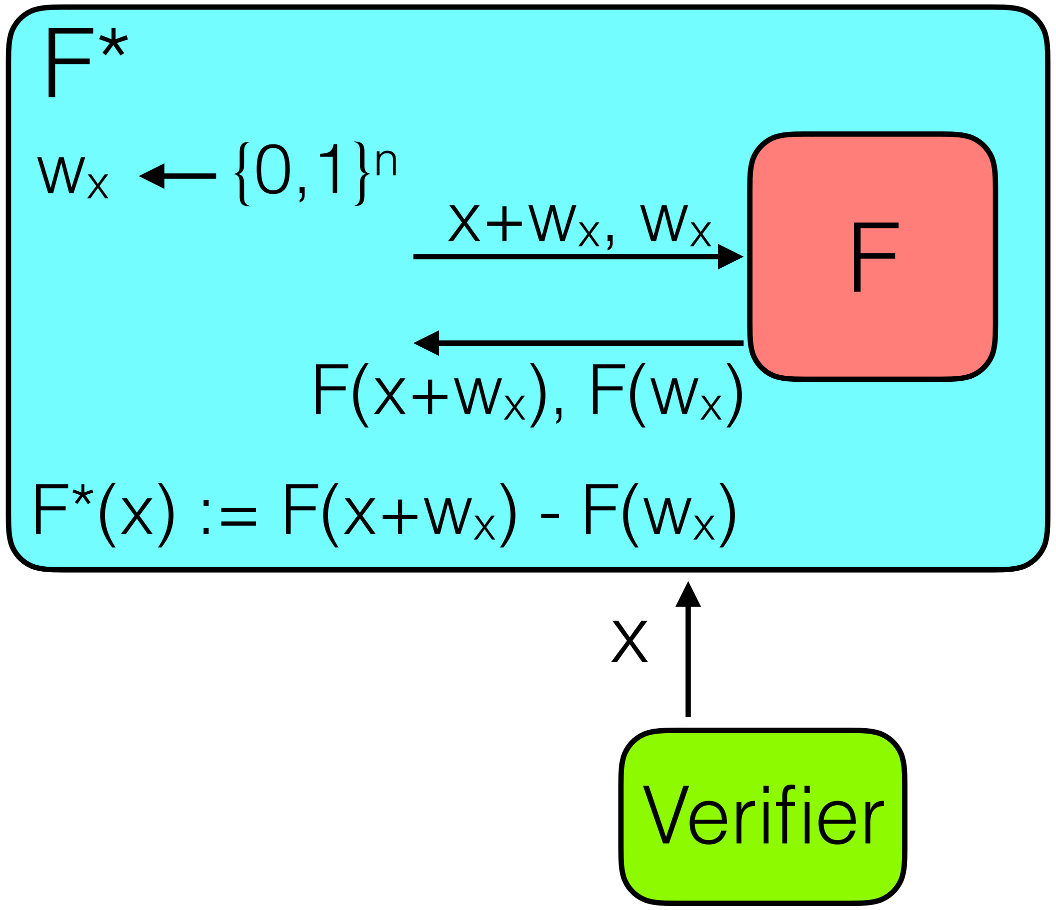

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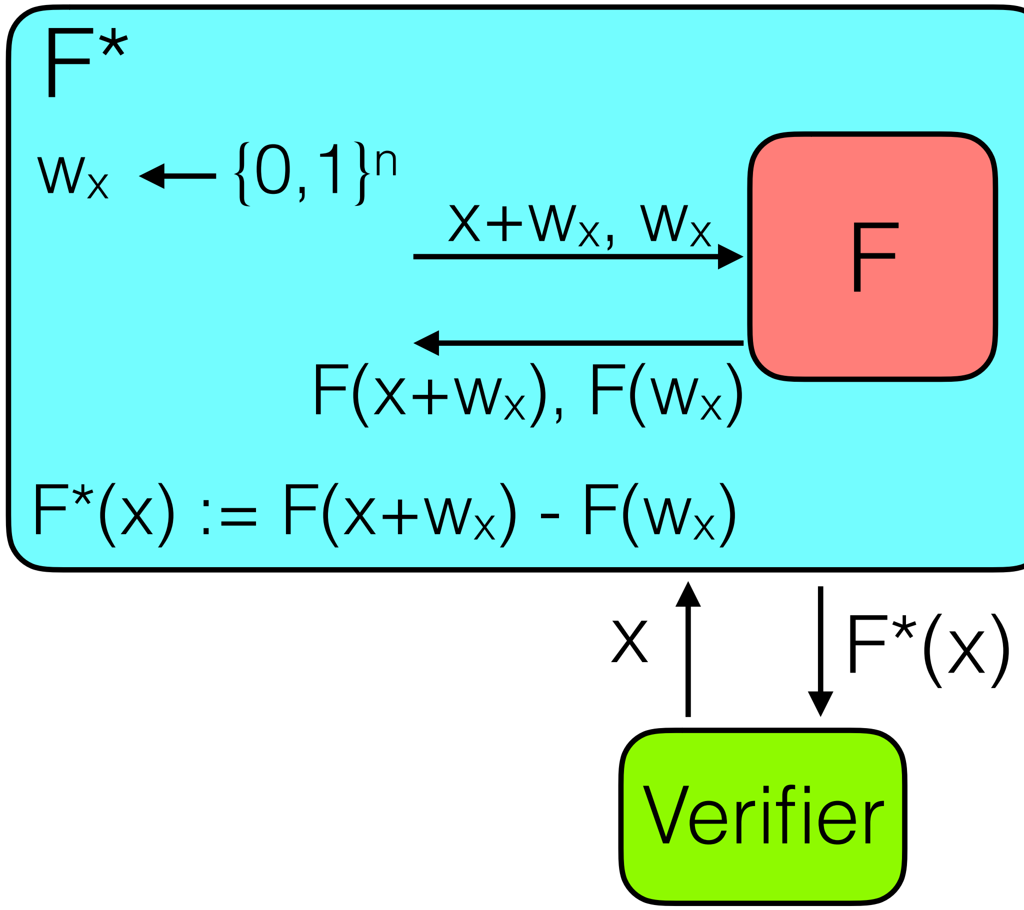

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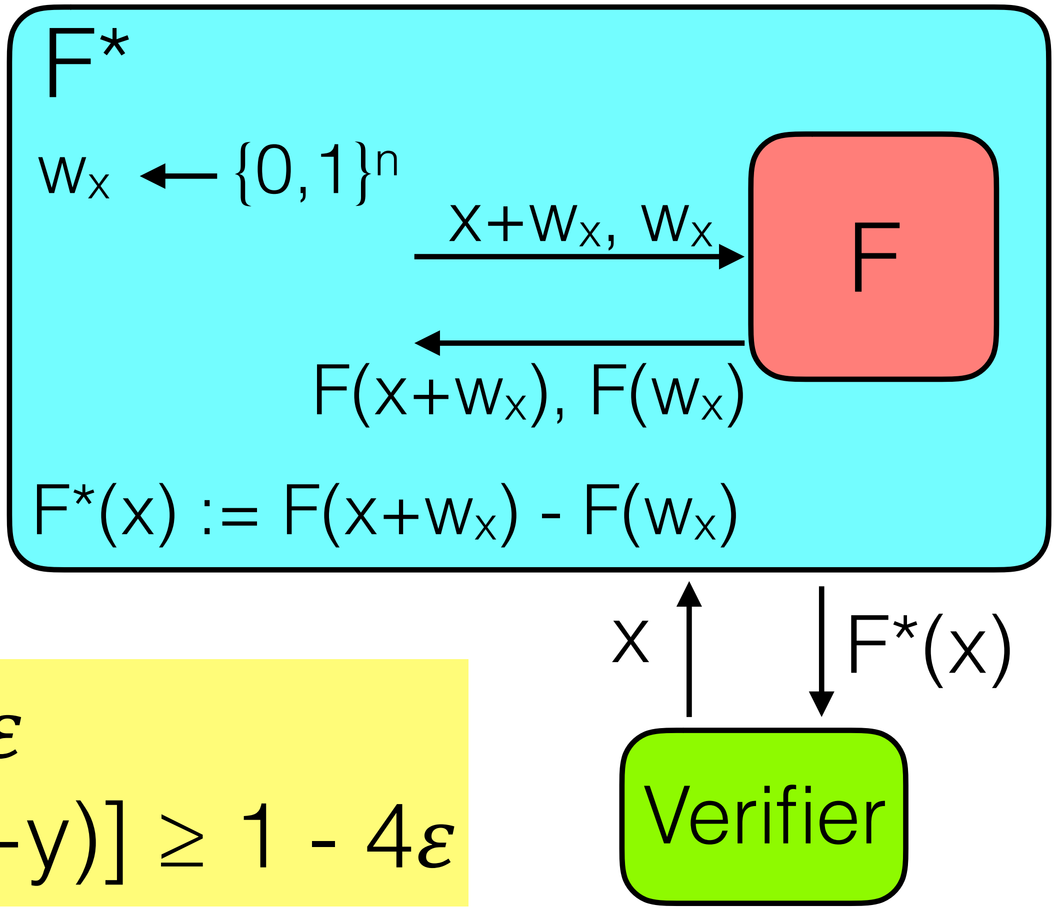

Average to worst case:

If $\Pr_{x,y,F}[F(x) + F(y) = F(x+y)] \geq 1 - \varepsilon$
then $\forall x,y,\ \Pr_{F^*}[F^*(x) + F^*(y) = F^*(x+y)] \geq 1 - 4\varepsilon$

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Quasi-distributions are essential.
Not just a technique! Can't state results without them.

Thanks!

full version available on ECCC
(TR18-067)

Non-Signaling Players

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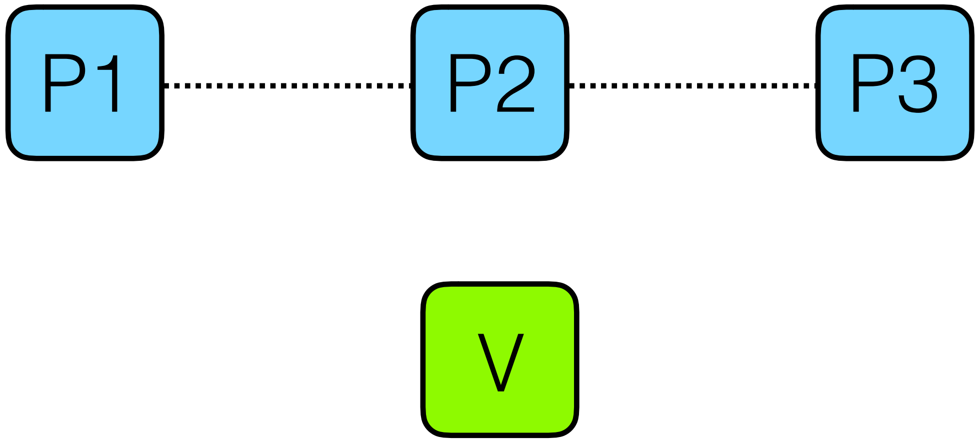

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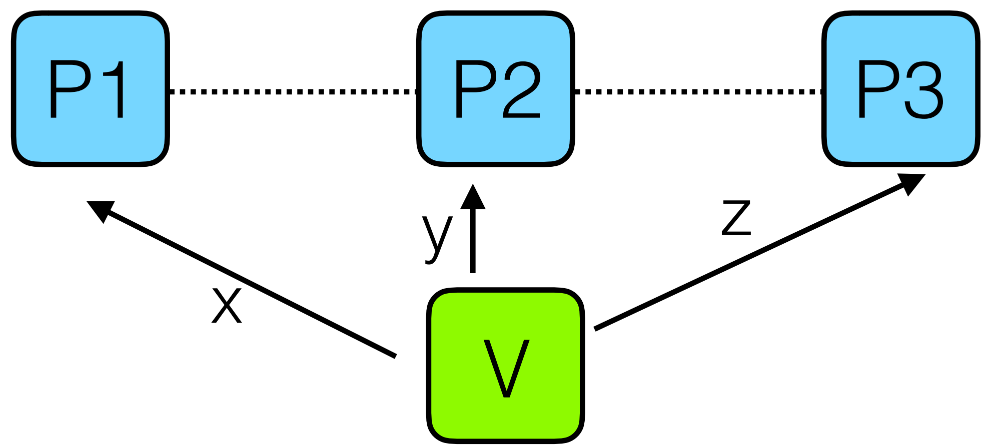

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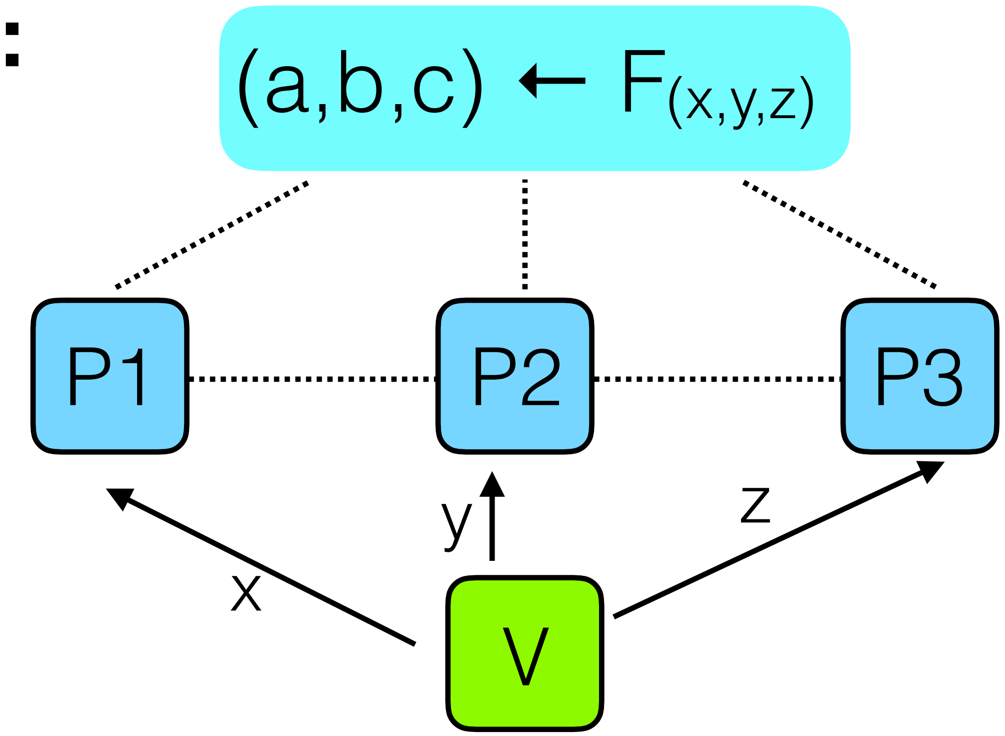

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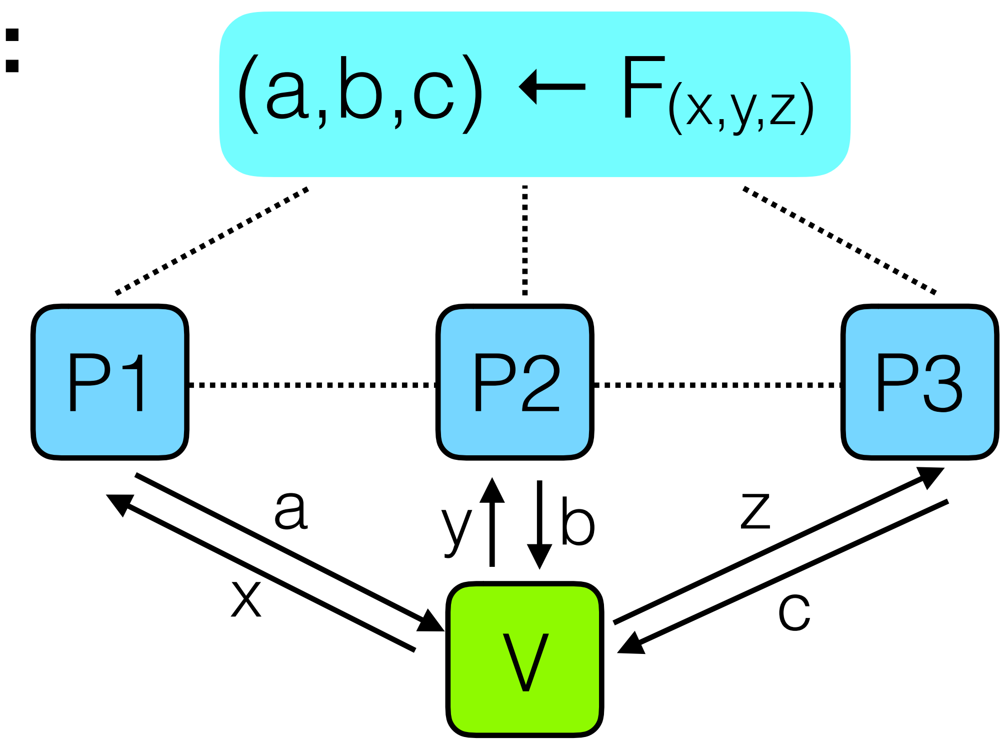

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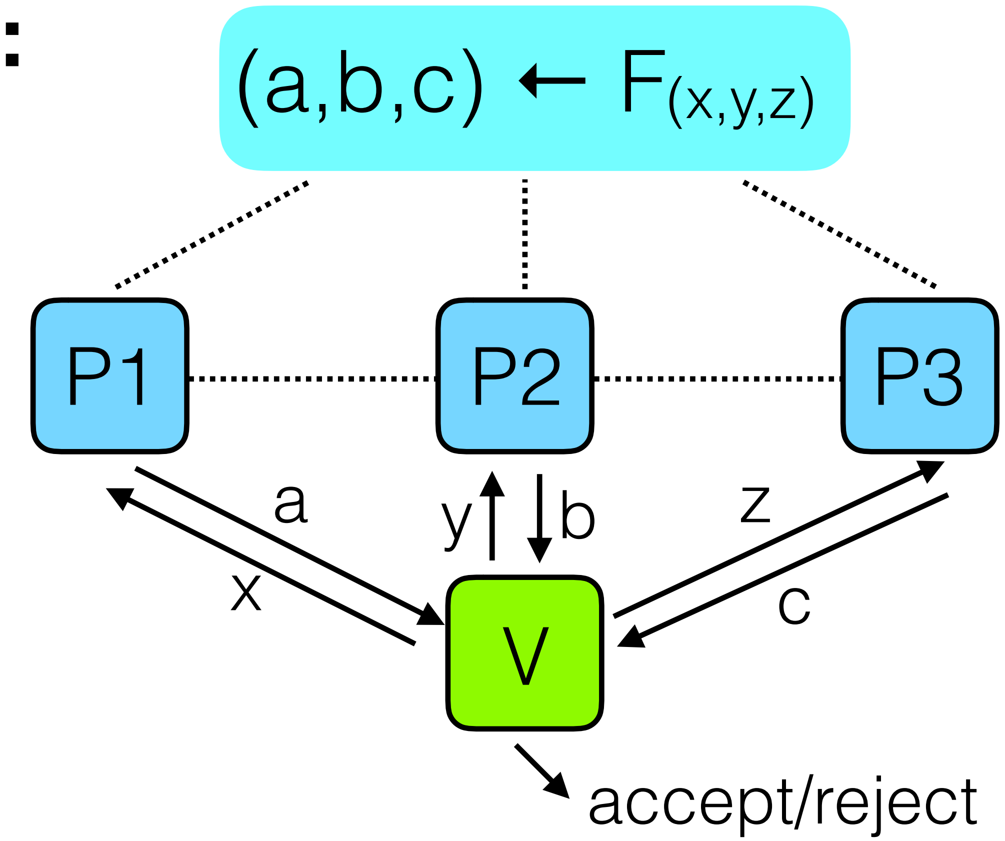

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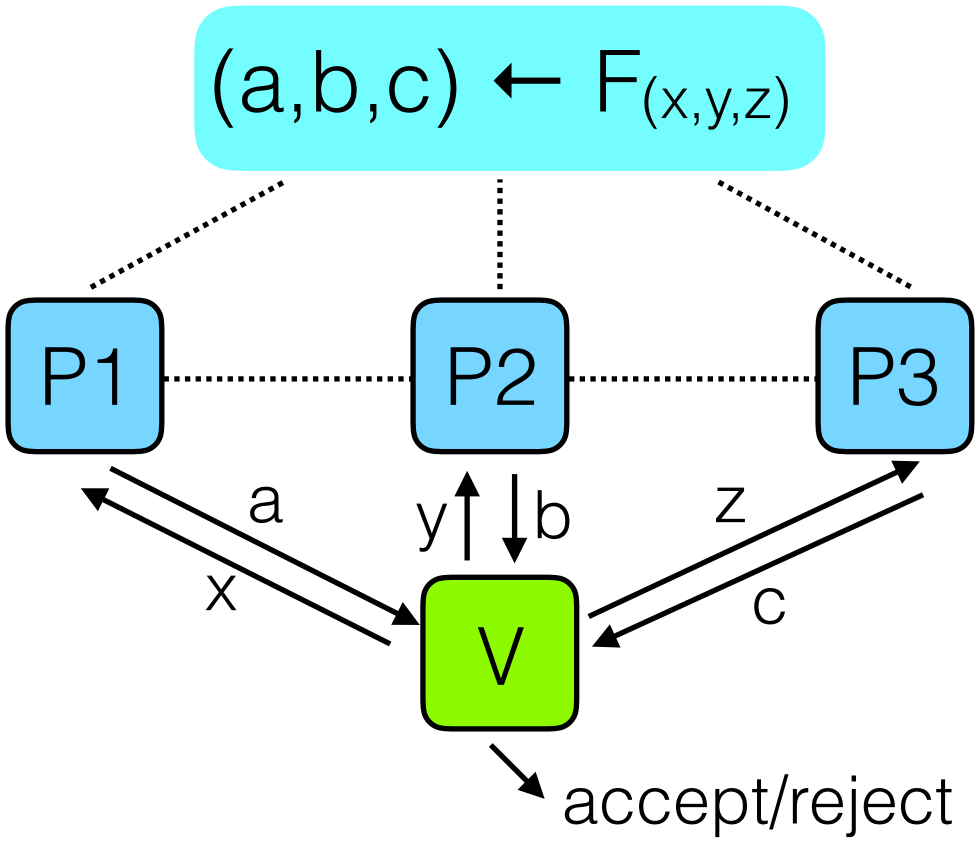
A k-non-signaling player P is a collection of distributions $\{F_{(x1, \ldots, xk)}\}$ over functions f: $[k] \rightarrow \{0,1\}$, $\forall$ tuples $(x_1, \ldots, x_k)$ that satisfies the **non-signaling property:**

$\forall (x_1, \ldots, x_k), (y_1, \ldots, y_k)$ where

$S = \{i : x_i = y_i\}$, then

$F_{(x1, \ldots, xk)} |_S \equiv F_{(y1, \ldots, yk)} |_S$



Analogues of all three theorems hold for ns players

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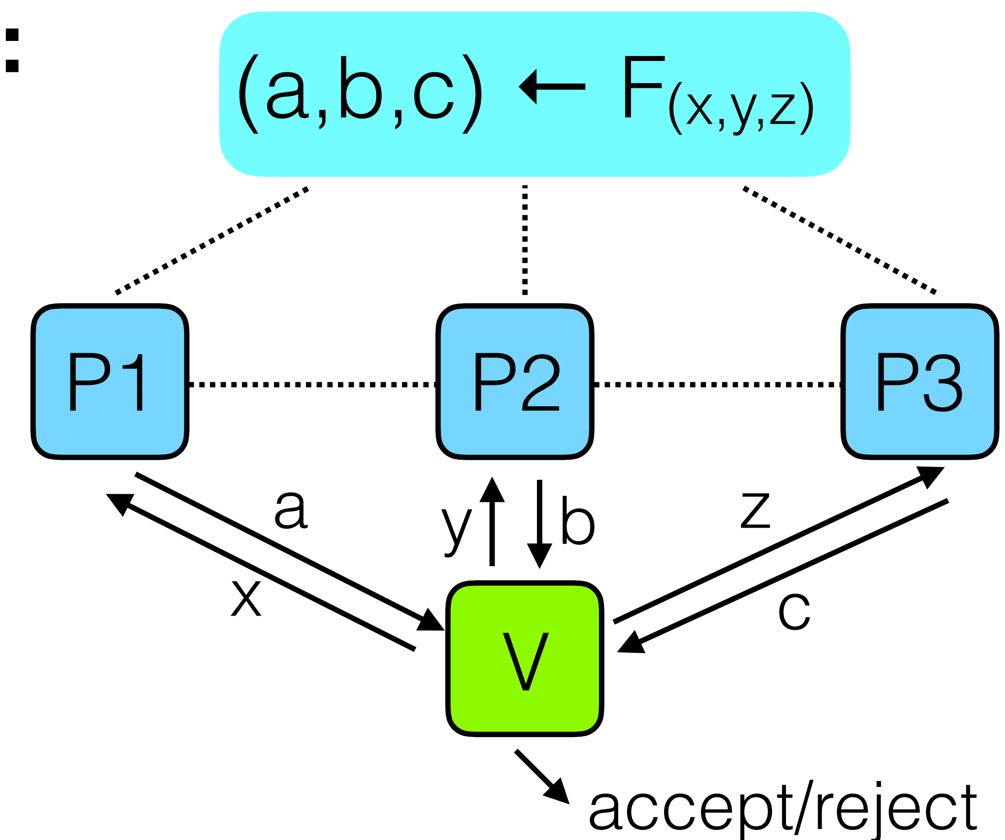

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Theorem 1 for players

k-non-signaling players ⟷ local quasi-distributions over k-tuples of f's