# Learning Fast Requires Good Memory: Time-Space Tradeoff Lower Bounds for Learning

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Based on joint works with:

**Sumegha Garg, Gillat Kol, Avishay Tal**

[R16, KRT17, R17, GRT18]

## This Talk:

**A line of recent works studies time-space (memory-samples) lower bounds for learning**

[S14, SVW16, R16, VV16, KRT17, MM17, R17, MM18, BOGY18, GRT18, DS18, AS18, DKS19, SSV19, GRT19, GKR19]

## Main Message:

**For some learning problems, access to a relatively large memory is crucial. In other words, in some cases, learning is infeasible, due to memory constraints**

## Original Motivation: Online Learning Theory:

**Initiated by:**

**[Shamir 2014], [Steinhardt-Valiant-Wager 2015]:**

**Can one prove unconditional lower bounds on the number of samples needed for learning, under memory constraints?**

(when each sample is viewed only once - also known as online learning)

## Example: Parity Learning:

$x = (x_1, \dots, x_n) \in_R \{0,1\}^n$ is unknown

A learner gets a stream of random linear equations (mod 2) in $x_1, \dots, x_n$, one by one, and tries to learn $x$

## Formally:

The learner gets a stream of samples: $(a_1, b_1), (a_2, b_2) \dots$ , where $\forall t$ :

$a_t \in_R \{0,1\}^n$ and $b_t = a_t \cdot x$ (inner product mod 2)

The learner needs to solve the equations and find $x$

(no noise)

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ is unknown

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$$a_1 = (1, 1, 0, 1, 1)\,, \qquad b_1 = 0$$

$$x_1 + x_2 + x_4 + x_5 = 0 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_2 = (0, 1, 1, 0, 0), \qquad b_2 = 0$$

$$x_2 + x_3 = 0 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_3 = (0, 0, 1, 1, 1)\,, \qquad b_3 = 0$$

$$x_3 + x_4 + x_5 = 0 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_4 = (0, 1, 1, 1, 0)\,, \qquad b_4 = 0$$

$$x_2 + x_3 + x_4 = 0 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_5 = (1, 1, 0, 0, 1)\,, \qquad b_5 = 0$$

$$x_1 + x_2 + x_5 = 0 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_6 = (0, 0, 1, 1, 0)\,, \qquad b_6 = 1$$

$$x_3 + x_4 = 1 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_7 = (0, 1, 0, 1, 1)\,, \qquad b_7 = 0$$

$$x_2 + x_4 + x_5 = 0 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_8 = (1, 0, 0, 0, 1)\,, \qquad b_8 = 1$$

$$x_1 + x_5 = 1 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_9 = (1, 1, 1, 1, 0)\,, \qquad b_9 = 0$$

$$x_1 + x_2 + x_3 + x_4 = 0 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_{10} = (0, 1, 1, 1, 1)\,, \qquad b_{10} = 1$$

$$x_2 + x_3 + x_4 + x_5 = 1 \pmod 2$$

# Ready to Play?

$x = (x_1, x_2, x_3, x_4, x_5)$ **is unknown**

$$a_{11} = (0, 0, 0, 0, 0)\,, \qquad b_{11} = 0$$

$$0 = 0 \pmod 2$$

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**A learner gets a stream of samples:** $(a_1, b_1), (a_2, b_2) \ldots$ **,**

**where** $\forall t$ **:** $a_t \in_R \{0,1\}^n$ **and** $b_t = a_t \cdot x$ **(mod 2)**

**and needs to solve the equations and find** $x$

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$O(n)$ **samples,** $O(n^2)$ **memory bits**

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**By trying all possibilities:**

$O(n)$ **memory bits, exponential number of samples**

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**memory bits or an exponential number of samples**
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(for memory of size $< n$, relatively easy to prove lower bounds, since inner product is a good two-source extractor)

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I will focus on super-linear lower bounds on the memory size

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**Best upper bound on the memory size :** $\approx \frac{n^2}{4}$

(when the number of samples is sub-exponential)

# Motivation: Machine Learning Theory:

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Very interesting to understand how much memory is needed for learning

Our result gives a concept class that can be efficiently learnt if and only if the learner has a quadratic-size memory

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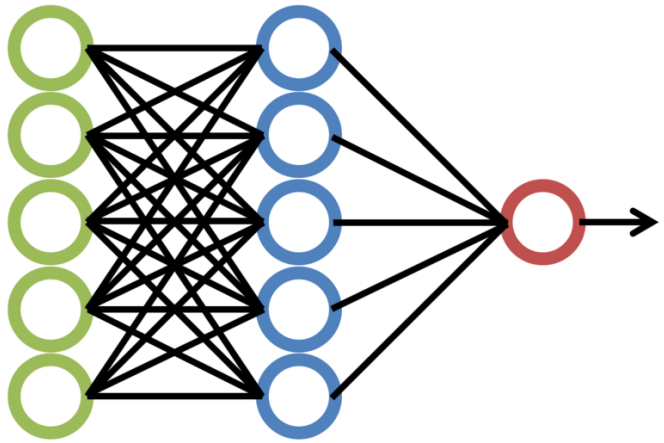
“Good” memory may be crucial in learning processes

# Example: Neural Networks

**Many learning algorithms try to learn a concept by modeling it as a neural network. The algorithm keeps in the memory some neural network and updates the weights when new samples arrive. The memory used is the size of the network**

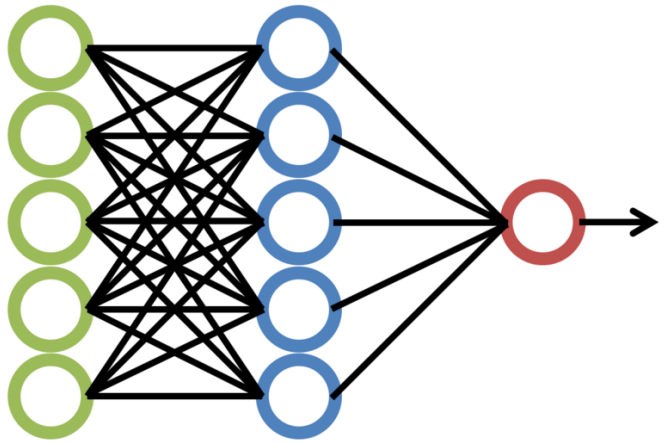


# Example: Neural Networks

**Many learning algorithms try to learn a concept by modeling it as a neural network. The algorithm keeps in the memory some neural network and updates the weights when new samples arrive. The memory used is the size of the network**

**Conclusion: such algorithms cannot learn certain concept classes, if each sample is viewed only once and the size of the neural network is not sufficiently large**

**For example, for learning parities, the memory size must be quadratic**



# Motivation: Bounded Storage Cryptography [Maurer 92]:

## [Maurer, CM, AR, ADR, Vadhan, DM,…]

**In the bounded storage model. Alice and Bob want to interact securely, in the presence of an adversary with bounded memory size**

Alice → Bob

## Applications to Bounded Storage Cryptography:

**[R16]:**

Secret Key Encryption/Decryption protocol

**[Guan-Zhandry 2019]:**

Key Agreement, Bit Commitment, Oblivious Transfer

**(all these protocols have advantages over previous works)**

## Motivation: Complexity Theory:

**Time-Space Lower Bounds for computing a function $f(x_1, .., x_n)$ have been studied for a long time, in various models [BJS 98, Ajt 99, BSSV 00, For 97, FLvMV 05, Wil 06,…]**

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These results proved lower bounds of at most $n^{1+\delta}$ on the time needed, under space constraints

How come we prove exponential bounds on the time, under space constraints?

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Lower bounds for computing $f(x_1, .., x_n)$ assume that $x_1, .., x_n$ can always be accessed (The inputs are stored for free)

In our case, after the learner saw $(a_t, b_t)$, the sample $(a_t, b_t)$ cannot be accessed again (unless stored in the memory)

## Motivation: Pseudo Randomness:

**Alternative way to construct pseudorandom generators for log-space, using $\mathrm{polylog}(n)$ truly-random bits**

**(doesn't match the best known generators that use only $\mathrm{O}(\log^2(n))$ bits [Nisan 90, INW 94, GR 14])**

**[Kol-R-Tal 2017]:**

**Learning sparse parities requires either super-linear memory size or a super-polynomial number of samples**

**Sparsity-$k$ parity learning: Same as parity learning but it is known in advance that at most $k$ of the variables $x_1, \dots, x_n$ are 1**

## [Kol-R-Tal 2017]:

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**For sparsity $k < n^{0.99}$: Any algorithm requires**

**1) $\Omega(n \cdot k)$ memory bits or $2^{\Omega(k)}$ samples**

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**Conclusion: Learning $\log(n)$-sparse parities, linear-size DNFs, linear-size CNFs, linear-size decision trees, $\log(n)$-size Juntas requires super-linear memory size or a super-polynomial number of samples**

**[R 17, MM 18, BOGY 18, GRT 18]:**

**For a large class of learning problems, any learning algorithm requires quadratic memory size or an exponential number of samples**

**[BOGY 18, GRT 18] build on [R 17]**

**[MM 18] builds on an earlier paper [MM 17] that obtained a similar result but with a linear lower-bound of $1.25 \cdot n$ on the memory size**

## [R 17, MM 18, BOGY 18, GRT 18]:

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[R 17, MM 18, BOGY 18, GRT 18]:

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I will focus on [R 17, GRT 18]

Additional follow-up works (building on [R 17]):

Memory-Sample lower bounds for:

linear regression with small error [SSV 19]

two-pass learning [GRT 19]

## A Learning Problem as a Matrix :

$A, X$ : finite sets

$M: A \times X \to \{-1, 1\}$ : a matrix

$x \in_R X$ is unknown. A learner tries to learn $x$ from a stream $(a_1, b_1), (a_2, b_2) \ldots$, where $\forall t$ :

$a_t \in_R A$ and

$b_t = M(a_t, x)$

$X$ : concept class ( $= \{0, 1\}^n$ in parity learning)

$A$ : possible samples ( $= \{0, 1\}^n$ in parity learning)

## Theorem [R 17], [Garg-R-Tal 18]:

**Assume that any submatrix of $M$ of fraction $2^{-k} \times 2^{-\ell}$ has bias of at most $2^{-r}$. Then, any learning algorithm requires either $\Omega(k \cdot \ell)$ memory bits or $2^{\Omega(r)}$ samples**

**In particular, for large classes of learning problems, any learning algorithm requires either memory of size $\Omega((\log|A|) \cdot (\log|X|))$ or an exponential number of samples**

(A new general proof technique. Implies all previous results)

(A related result by Beame, Oveis-Gharan, Yang (building on [R 17]))

## Applications: (examples)

**Learning from low-degree equations:** A learner tries to learn $x = (x_1, \ldots, x_n) \in_R \{0,1\}^n$, from random multilinear polynomial equations of degree at most $d$ (over $F_2$):

Requires $\Omega(n^{d+1})$ memory or $2^{\Omega(n)}$ samples

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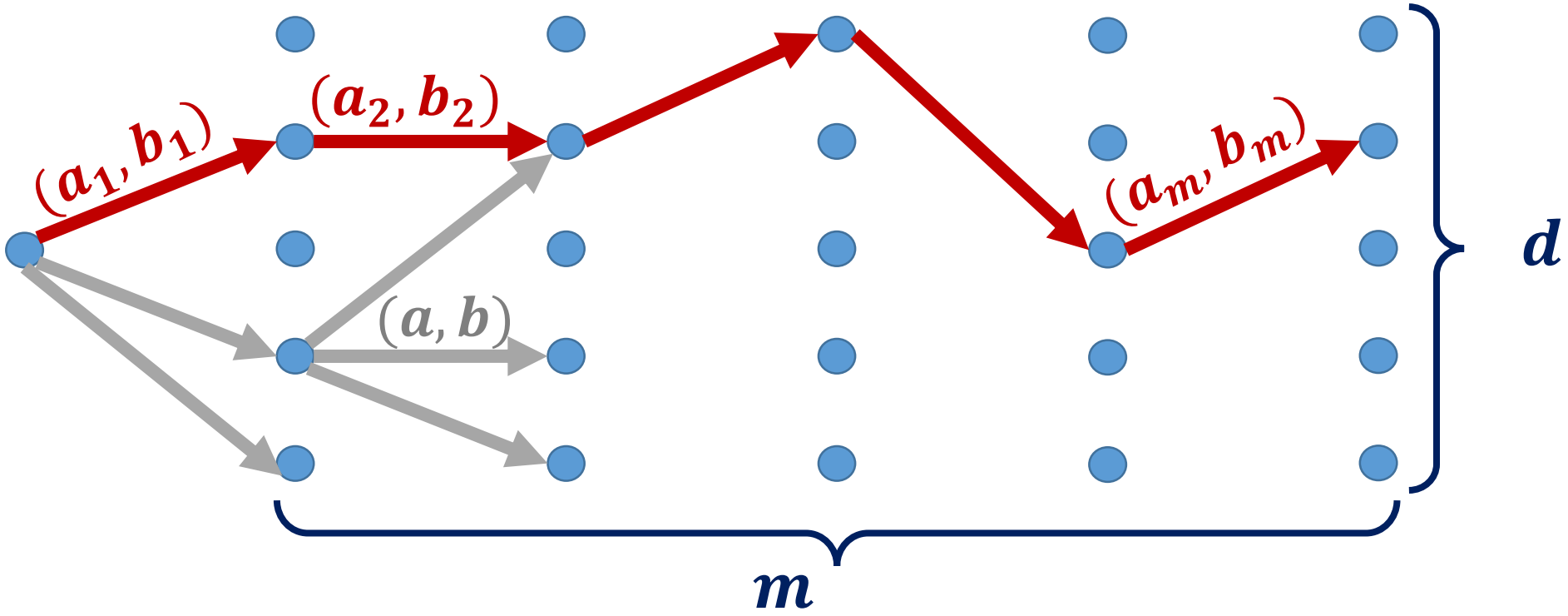
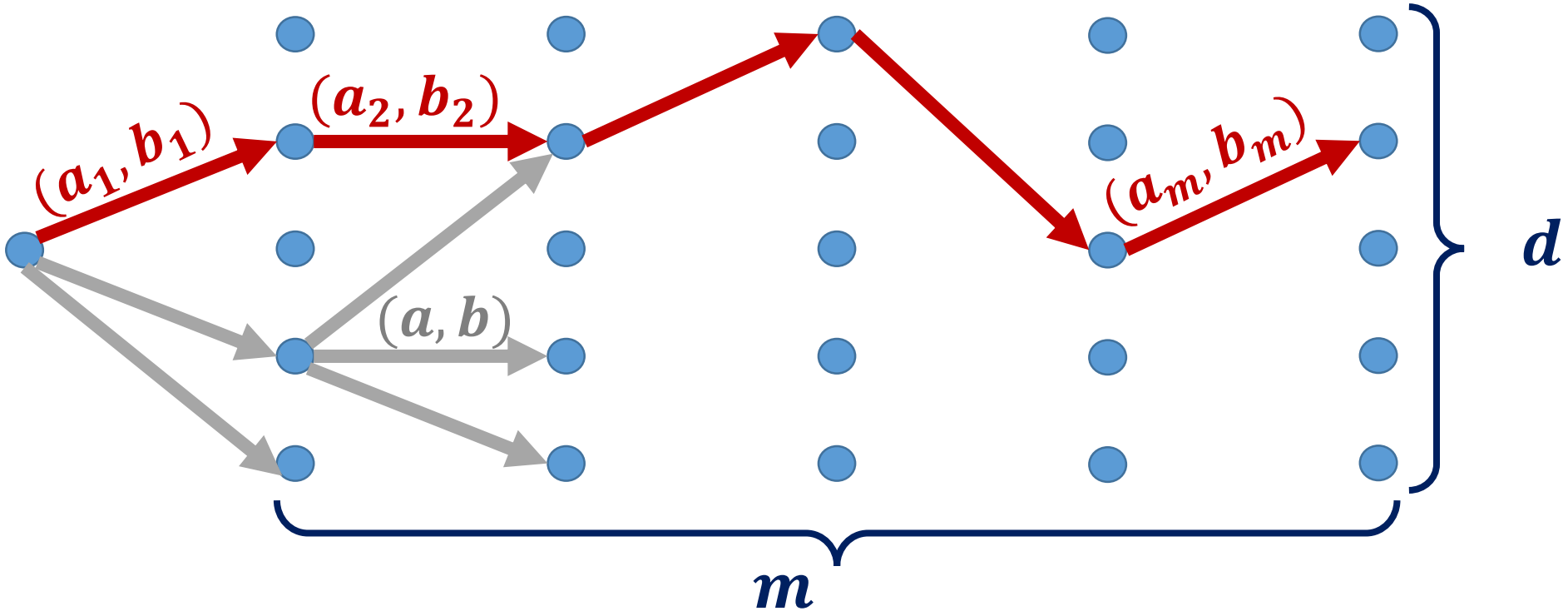
**Error correcting codes…** **Random matrices…**

# Branching Program (length $m$, width $d$): (for parity learning)



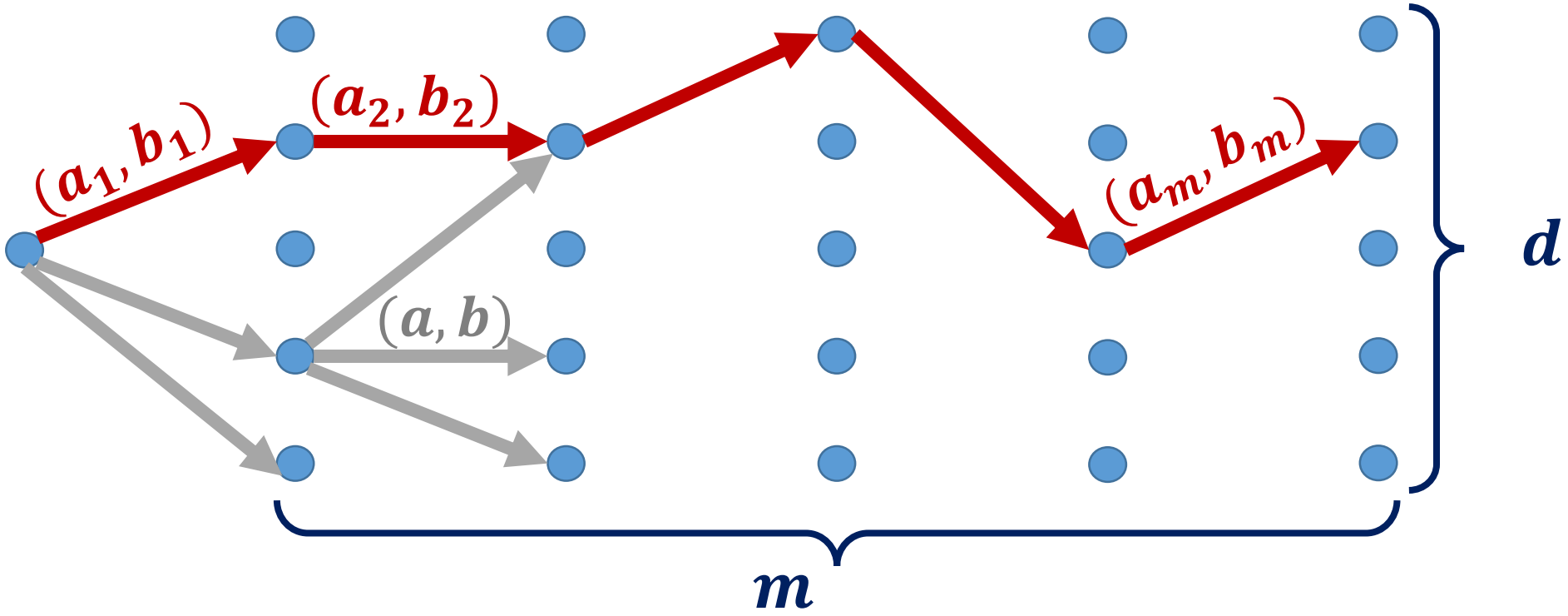

Each layer represents a time step. Each vertex represents a memory state of the learner. Each non-leaf vertex has $2^{n+1}$ outgoing edges, one for each $(a, b) \in \{0,1\}^n \times \{-1, 1\}$

# Branching Program (length $m$, width $d$): (for parity learning)




The samples $(a_1, b_1), .., (a_m, b_m)$ define a computation-path. Each vertex $v$ in the last layer is labeled by $\hat{x}_v \in \{0,1\}^n$. The output is the label $\hat{x}_v$ of the vertex reached by the path

# Branching Program (length $m$, width $d$): (for parity learning)




**Example: BP for Parity learning: Any BP with width $d \leq 2^{n^2/20}$ and length $m \leq 2^{n/100}$, outputs the correct $x$ with exponentially small probability**

# Proof Outline (for parity): [R17]

**same proof techniques was used in several follow-up works:**
[BOGY18, GRT18, SSV19, GRT19]

# Interesting Idea in the Proof: (very high level)

Significant vertices: $v$ s.t. conditioned on the event that the computation-path reaches $v$, $x$ can be guessed with non-negligible probability

$Pr(v)$ = probability that the computation-path reaches $v$

We want to prove: If $v$ is significant, $Pr(v) \leq 2^{-\Omega(n^2)}$

Hence, there are at least $2^{\Omega(n^2)}$ significant vertices

$B$ = the event that some "atypical" things happen

We show that $Pr(B) \leq 2^{-\Omega(n)}$ (but much larger than $2^{-\Omega(n^2)}$)

We show that if $v$ is significant, $Pr(v|\overline{B}) \leq 2^{-\Omega(n^2)}$

## Proof Outline:

$T$ = same as the computation path, but stops when "atypical" things happen (stopping rules). All definitions are with respect to $T$

$Pr(T \text{ stops})$ is exp small (but much larger than $2^{-\Omega(n^2)}$)

$\mathbf{P}_{x|v}$ = distribution of $x$ conditioned on the event that the path $T$ reaches $v$

Significant vertices: $v$ s.t. $||\mathbf{P}_{x|v}||_2 \geq 2^{\delta n} \cdot 2^{-n}$

$Pr(v)$ = probability that the path $T$ reaches $v$

We prove: If $v$ is significant, $Pr(v) \leq 2^{-\Omega(n^2)}$

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## Proof Outline:

If $s$ is significant, $Pr(s) \leq 2^{-\Omega(n^2)}$

Progress Function: For layer $L_i$,

$$Z_i = \sum_{v \in L_i} Pr(v) \cdot \langle \mathbf{P}_{x|v} , \mathbf{P}_{x|s} \rangle^{\delta n}$$

1) $Z_0 = 2^{-2\delta n^2}$
2) $Z_i$ is very slowly growing: $Z_0 \approx Z_m$
3) If $s \in L_m$, then $Z_m \geq Pr(s) \cdot 2^{(\delta n)^2} \cdot 2^{-2\delta n^2}$

Hence: If $s$ is significant, $Pr(s) \leq 2^{-(\delta n)^2} = 2^{-\Omega(n^2)}$

(the hard step is step 2)

## How we prove that $Z_i$ is very slowly growing:

$$Z_i = \sum_{v \in L_i} Pr(v) \cdot \langle \mathbf{P}_{x|v} , \mathbf{P}_{x|s} \rangle^{\delta n}$$

$$Z_{i+1} = \sum_{v \in L_{i+1}} Pr(v) \cdot \langle \mathbf{P}_{x|v} , \mathbf{P}_{x|s} \rangle^{\delta n}$$

$$Z' = \sum_{e:\, L_i \to L_{i+1}} Pr(e) \cdot \langle \mathbf{P}_{x|e} , \mathbf{P}_{x|s} \rangle^{\delta n}$$

**By a simple convexity argument,** $Z_{i+1} \leq Z'$

**The hard part is to show that** $Z'$ **is only negligibly larger than** $Z_i$

## How we prove that $Z'$ is only negligibly larger than $Z_i$:

$$Z_i = \sum_{v \in L_i} Pr(v) \cdot \langle \mathbf{P}_{x|v} , \mathbf{P}_{x|s} \rangle^{\delta n}$$

$$Z' = \sum_{e:\, L_i \to L_{i+1}} Pr(e) \cdot \langle \mathbf{P}_{x|e} , \mathbf{P}_{x|s} \rangle^{\delta n}$$

**We show that, on average,**

$$\sum_{e:\, v \to L_{i+1}} Pr(e) \cdot \langle \mathbf{P}_{x|e} , \mathbf{P}_{x|s} \rangle^{\delta n}$$

**is only negligibly larger than** $Pr(v) \cdot \langle \mathbf{P}_{x|v} , \mathbf{P}_{x|s} \rangle^{\delta n}$

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is only negligibly larger than $Pr(v) \cdot \langle \mathbf{P}_{x|v}\, , \mathbf{P}_{x|s} \rangle^{\delta n}$

Roughly speaking: For parity:

$\langle \mathbf{P}_{x|e}\, , \mathbf{P}_{x|s} \rangle$ are close, up to a normalization, to the Fourier coefficients of $\mathbf{P}_{x|v} \cdot \mathbf{P}_{x|s}$. By introducing stopping rules for the path $T$, we are able to bound the $L_2$-norm of $\mathbf{P}_{x|s}$ and the $L_\infty$-norm of $\mathbf{P}_{x|v}$ and hence the $L_2$-norm of $\mathbf{P}_{x|v} \cdot \mathbf{P}_{x|s}$, so that the Fourier coefficients of $\mathbf{P}_{x|v} \cdot \mathbf{P}_{x|s}$ are small on average. Another stopping rule ensures that the normalization doesn't distort by much

## Stopping Rules:

Stop on a vertex $v$ if:

1) $||\mathbf{P}_{x|v}||_2$ is large
2) $\mathbf{P}_{x|v}(x)$ is large
3) The next edge corresponds to a large Fourier coefficient of $\mathbf{P}_{x|v}$

The stopping rules do not depend on $s$

They do depend on $x$

## Summary:

**For a large class of learning problems, any learning algorithm requires either super-linear memory size or a super-polynomial number of samples**

## Main Message:

**For some learning problems, access to a relatively large memory is crucial. In other words, in some cases, learning is infeasible, due to memory constraints**

[S14, SVW16, R16, VV16, KRT17, MM17, R17, MM18, BOGY18, GRT18, DS18, AS18, DKS19, SSV19, GRT19, GKR19]

# Thank You!