HARDNESS OF FUNCTION COMPOSITION FOR SEMANTIC READ ONCE BRANCHING PROGRAMS

Jeff Edmonds, Venkatesh Medabalimi, Toniann Pitassi

June 23, 2018

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LOWER BOUNDS
AGAINST
FUNCTION
COMPOSITION

P AND L



P: Polynomial time computable functions.

* * * *

 $L: Functions \ computable \ in \ logarithmic \ space.$

 $L \overset{?}{\subset} \textbf{P}$

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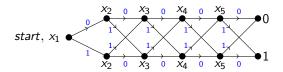
Branching Programs

$$f(x_1, x_2, ..., x_n) \to \{0, 1\}$$
 $x_i \in \{0, 1\}, \forall i \in [n]$

DEFINITION

Deterministic Branching program

- DAG with a source node and two sinks, 1-sink (for accept) and 0-sink (for reject).
- Each non-sink node is labeled by some x_i , outdegree 2 with an edge each for $x_i = 0$ and $x_i = 1$.



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Non-det Branching Programs

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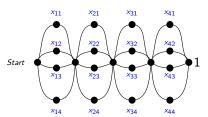
LOWER BOUND AGAINST FUNCTION

OPEN PROBLEMS

DEFINITION

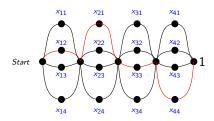
Non-deterministic Branching program (NBP)

 allow unlabelled guessing nodes and arbitrary out-degree.



The size of a NBP= number of labelled nodes.

NBP COMPUTING $f: \{0,1\}^n \to \{0,1\}$



 $f(u)=1 \iff \exists$ a path from source to accept node that is consistent with input u.

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PROGRAM SIZE AND SPACE COMPLEXITY OF COMPUTING F

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Open Problems

•
$$BP(f_n) = \min_{B \in BP \text{ computing } f_n} \text{ size } (B)$$

•
$$S(f_n) = \min_{T \in \text{ non-uniform TMs computing } f_n} \text{ space complexity } (T)$$

•
$$\log(BP(f_n)) \approx S(f_n)$$
 [Cobham '66]

BIG PICTURE

• It is easy to show functions with high $BP(f_n)$ exist.

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BIG PICTURE

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LOWER BOUNDS AGAINST FUNCTION

- It is easy to show functions with high $BP(f_n)$ exist.
- Can we show that some function in P requires exponential size BP?
 - amounts to showing $L \subset P$.

BPs and other Computation Models

Formulas BranchingPrograms Circuits $L(f) \geq BP(f) \geq \frac{1}{3}C(f)$

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BPs and other Computation Models

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Draw Dropy mag

Formulas BranchingPrograms Circuits $L(f) \geq BP(f) \geq \frac{1}{3}C(f)$

Nechiporuk

 $\Omega\left(\frac{n^2}{\log^2 n}\right)$

 $\Omega(n^3)$

Random Restrictions

Gate Elimination

 $\Omega(n)$

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OPEN PROBLEMS

 Bounded Width: same as NC¹, Barrington's characterization.

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RECTANGLES

LOWER BOUNDS

AGAINST

AGAINST FUNCTION COMPOSITION

- Bounded Width: same as NC¹, Barrington's characterization.
- Length Restricted: give Time-Space tradeoffs.

TIME-SPACE TRADEOFFS

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OPEN PROBLEMS

•
$$t \le cn \Longrightarrow s = 2^{\Omega(n)}$$

Jukna'09

TIME-SPACE TRADEOFFS

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OPEN PROBLEMS

• $t \le cn \Longrightarrow s = 2^{\Omega(n)}$

- Jukna'09
- culmination results by Ajtai '99 and Beame, Jayram,
 Saks '01

TIME-SPACE TRADEOFFS

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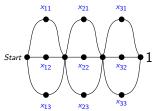
OPEN PROBLEMS

- $t \le cn \implies s = 2^{\Omega(n)}$ Jukna'09
- culmination results by Ajtai '99 and Beame, Jayram, Saks '01
- We look at:

time-space tradeoffs for iterated function composition.

READ ONCE

• *Syntactic* read once: Along any path from source to sink any variable appears atmost once.



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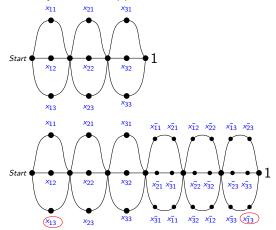
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RECTANGLES

LOWER BOUNDS
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READ ONCE

• *Syntactic* read once: Along any path from source to sink any variable appears atmost once.



• Semantic read once: Along any consistent path from source to sink no variable is read more than once.

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AGAINST FUNCTION COMPOSITION

SYNTACTIC WEAKER THAN SEMANTIC

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RESTRICTED: READ-ONCE

The Exact Perfect matching function (EPM_n) : accept a matrix iff it is a permutation matrix.

Jukna and Razborov '98 showed

THEOREM

Every syntactic read once NBP computing EPM_n must have size $2^{\Omega(n)}$

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OPEN PROBLEMS

THEOREM (JUKNA)

 EPM_n can be solved by a semantic read once NBP of size $O(n^3)$.

 $\left[\begin{array}{ccc}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{array}\right]$

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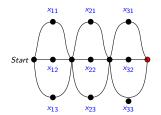
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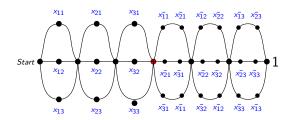
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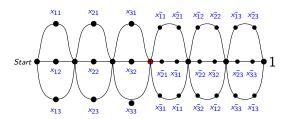
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Sees only 1s

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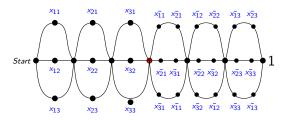
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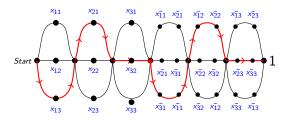
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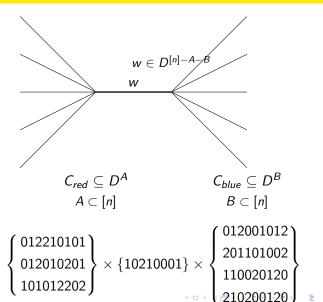
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LOWER BOUNDS AGAINST FUNCTION



EMBEDDED RECTANGLES

$$C_{red} \times \{w\} \times C_{blue} \subseteq [D]^n$$



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LOWER BOUNDS
AGAINST
FUNCTION
COMPOSITION

EMBEDDED RECTANGLES

$$C_{red} \times \{w\} \times C_{blue} \subseteq [D]^n$$

 $w \in D^{[n]-A}$ W $C_{blue} \subseteq D^B$ $C_{red} \subseteq D^A$ $A \subset [n]$ $B \subset [n]$ 012001012 012210101) 201101002 012010201 $\times~\{102101\}~\times$ 110020120

210200120

Hardness of Function Composition for Semantic Read once Branching Programs

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RECTANGLES

Lower Bounds against Function

- an approach to separating NC¹ from NC².
- KRW conjecture that for every random f and $\forall g$, $D(f \circ g) \ge \epsilon D(f) + D(g)$.
- KRW conjecture on formula size of a composed function fog.

$$L(fog) \approx L(f)L(g)$$
?

Understanding Composition

• How does space compose ?

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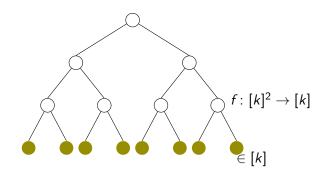


FIGURE: *TEP*⁴ that is height 4, degree 2.



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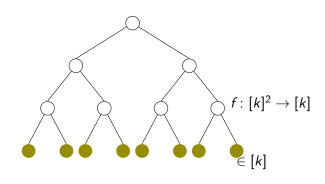


FIGURE: TEP_2^4 that is height 4, degree 2.

Is
$$BP(TEP_2^h) = \Omega(k^h)$$
 ??

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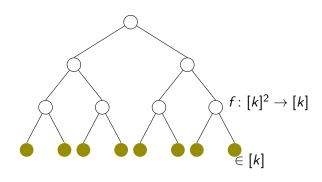


FIGURE: TEP_2^4 that is height 4, degree 2.

Is
$$BP(TEP_2^h) = \Omega(k^h)$$
 ?? $\Longrightarrow L \subset P$

BOOLEAN TEP

FUNCTION
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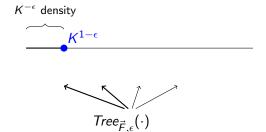
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LOWER BOUND FOR COMPOSITION

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PROGRAM

LOWER BOUNDS AGAINST FUNCTION COMPOSITION

COMPOSITION PROOF OVERVIEW A SPECIAL QUERY

STATE FOR EACH INPUT SPECIAL LOW ENTROPY NODE IN

ENTROPY NODE IN THE TREE

i wo-way Produ Sets Conclusion

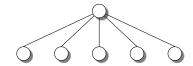
OPEN PROBLEMS

THEOREM

For any h, and k sufficiently large, there exists ϵ and \vec{F} such that any k-ary nondeterministic semantic read-once branching program for ternary $\text{Tree}_{\vec{F},\epsilon}$ requires size at least

$$\left(\frac{k}{\log k}\right)^h$$

BLACK WHITE PEBBLING UPPERBOUND



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FUNCTION

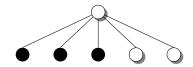
PROOF OVERVIEW

A SPECIAL QUERY
STATE FOR EACH
INPUT
SPECIAL LOW

ENTROPY NODE IN THE TREE TWO-WAY PRODUCT SETS

Conclusion

$\frac{1}{2}(d-1)h+1$ PEBBLES AT THIS MOMENT



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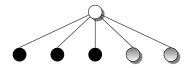
PROOF OVERVIEW

A SPECIAL QUERY
STATE FOR EACH
INPUT

SPECIAL LOW ENTROPY NODE IN THE TREE

TWO-WAY PRODUCT SETS CONCLUSION

GUESS THE REMAINING SIBLINGS



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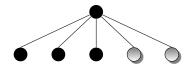
B A SPECIAL QUERY
STATE FOR EACH
INPUT

SPECIAL LOW ENTROPY NODE IN THE TREE

Two-way Product Sets Conclusion

Open Problems

INFER THE ROOT



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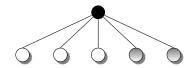
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STATE FOR EACH INPUT SPECIAL LOW ENTROPY NODE IN

ENTROPY NODE IN THE TREE TWO-WAY PRODUCT SETS

Conclusion

Unperble blacks



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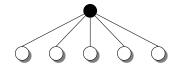
3 A SPECIAL QUERY
STATE FOR EACH
INPUT

SPECIAL LOW ENTROPY NODE IN THE TREE

TWO-WAY PRODUCT SETS CONCLUSION

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VERIFY GUESSES



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PROOF OVERVIEW

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ENTROPY NODE IN THE TREE TWO-WAY PRODUCT

TWO-WAY PRODUC SETS CONCLUSION

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LOWER BOUND USES INVERTIBLE FUNCTIONS

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COMPOSITION
PROOF OVERVIEW

A SPECIAL QUERY

STATE FOR EACH INPUT SPECIAL LOW

PECIAL LOW NTROPY NODE : HE TREE

TWO-WAY PRODUCT SETS

OPEN PROBLEMS

DEFINITION

A Latin Cube is a function $f : [k]^3 \to [k]$ such that f is invertible in each of its coordinates.

Equivalently, every element of [k] appears exactly once along every row, column and leg in the cube $[k]^3$.

4-INVERTIBLE FUNCTION, $f: [k]^3 \to [k]$

DEFINITION

Any element in [k] appears at most 4 times along any row, column or leg in the cube $[k]^3$.

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LOWER BOUNDS AGAINST FUNCTION

COMPOSITION

PROOF OVERVIEW

A SPECIAL QUERY
STATE FOR EACH

INPUT
SPECIAL LOW
ENTROPY NODE IN
THE TREE

THE TREE
TWO-WAY PRODUCT
SETS

ONCLUSION

Open Problems

PROOF OVERVIEW

• A small BP for a $Tree_{\vec{F.\epsilon}}$

 \Longrightarrow

Tree $_{\vec{F},\epsilon}$ accepts a large rectangle of inputs over its leaves

HARDNESS OF FUNCTION COMPOSITION FOR SEMANTIC READ ONCE BRANCHING

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MOTIVATION

Branching Programs

LOWER BOUNDS
AGAINST
FUNCTION

COMPOSITION
PROOF OVERVIEW

∃ A SPECIAL QUERY STATE FOR EACH INPUT

SPECIAL LOW ENTROPY NODE IN THE TREE

Two-way Product Sets Conclusion

PROOF OVERVIEW

HARDNESS OF FUNCTION COMPOSITION FOR SEMANTIC READ ONCE BRANCHING PROGRAMS

• A small BP for a $Tree_{\vec{F},\epsilon}$

 \Longrightarrow $Tree_{\vec{F,\epsilon}}$ accepts a large rectangle of inputs over its leaves

 A large rectangle over leaves \Rightarrow \exists a special node v* in the tree whose F_{v*} can be described in few bits.

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• A small BP for a $Tree_{\vec{F},\epsilon}$

 \Longrightarrow $Tree_{\vec{F,\epsilon}}$ accepts a large rectangle of inputs over its leaves

 A large rectangle over leaves \Rightarrow \exists a special node v* in the tree whose F_{v*} can be described in few bits.

• Show that the distribution on \vec{F} is rich or sufficiently random looking that one cannot save these bits.

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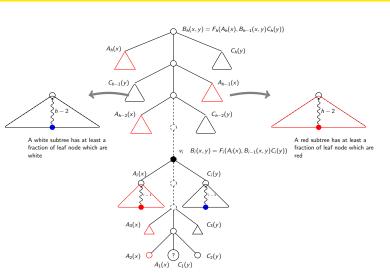
B A SPECIAL QUERY
STATE FOR EACH
INPUT

PECIAL LOW NTROPY NODE IN THE TREE

TWO-WAY PRODUCT SETS



FOR EVERY ACCEPTING INPUT \exists A SPECIAL QUERY STATE:



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3 A SPECIAL QUERY
STATE FOR EACH
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SPECIAL LOW

SPECIAL LOW ENTROPY NODE IN THE TREE

TWO-WAY PRODUCT SETS

FEW SPECIAL STATES $\Longrightarrow \exists$ A LARGE EMBEDDED RECTANGLE OVER LEAVES

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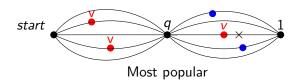
Branching Programs

> LOWER BOUNI AGAINST FUNCTION COMPOSITION

∃ A SPECIAL QUERY STATE FOR EACH INPUT SPECIAL LOW

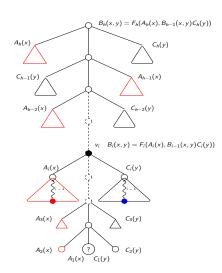
PECIAL LOW INTROPY NODE IN THE TREE

Two-way Product Sets Conclusion



- Choose a popular labelled path down the tree.
- Choose a popular red variable for the first red-subtree.
 Prune the input set. Continue to choose h red variables one for each red-subtree. Similarly for each blue-subtree.
- Fix the remaining variables in [n]-Red-Blue to the most popular projection 'w'.

∃ A LARGE EMBEDDED RECTANGLE OVER THE LEAVES.



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ODEN PRODURMS

\exists A NODE v* AT WHICH LEAVES IN BOTH RED AND BLUE TREES TAKE A LOT OF VALUES.

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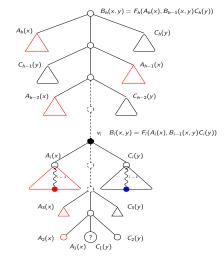
COMPOSITION
PROOF OVERVIE

∃ A SPECIAL QUERY STATE FOR EACH INPUT

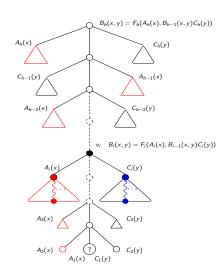
SPECIAL LOW ENTROPY NODE IN THE TREE

TWO-WAY PRODUCT SETS

Open Door ma



\exists A NODE v* AT WHICH BOTH RED AND BLUE CHILD TAKE A LOT OF VALUES.



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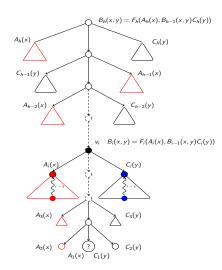
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INPUT
SPECIAL LOW
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THE TREE

TWO-WAY PRODUCT SETS CONCLUSION

\exists A NODE v* WITH LOW ENTROPY ON A TWO-WAY PRODUCT SET



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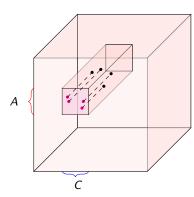
PROOF OVERVIEW

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Two-way Product Sets Conclusion

TWO-WAY PRODUCT SET AT v*, IN $F_{v*}()$



 $A, C \subset [k]$

$$|A| = |C| = r << k$$

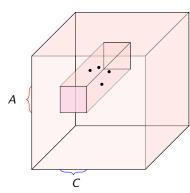
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∃ A SPECIAL QUERY STATE FOR EACH

ENTROPY NODE IN THE TREE

TWO-WAY PRODUCT Sets CONCLUSION

TWO-WAY PRODUCT SET AT v*, IN $F_{v*}()$



$$A, C \subset [k]$$

$$|A| = |C| = r << k$$

$$S_r = \{(x, Q(x, y), y) | x \in A, y \in C\}$$

$$|S| = r^2$$

FUNCTION
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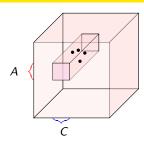
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TWO-WAY PRODUCT SETS

Conclusio

Open Problems

ENROPY OF SPREAD ON TWO-WAY PRODUCT SET CAN'T BE LOW



$$A, C \subset [k]$$

$$|A| = |C| = r << k$$

$$S_r = \{(\mathbf{x}, B(\mathbf{x}, \mathbf{y}), \mathbf{y}) | \mathbf{x} \in A, \mathbf{y} \in C\}$$

$$|S|=r^2$$

orall Two-way Product Sets S_r and target set \mathcal{T}_ϵ

$$\Pr_{f \sim \textit{U(All 4-invertible cubes})}[f(\textit{S}_r) \subset \textit{T}_{\epsilon}] \leq \frac{1}{k^{\epsilon r^2}}$$

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Conclusion

LABEL

Jeff Edmonds, Venkatesh MEDABALIMI, α^h Toniann Pitassi β_h α^{h-1} β^{h-1} β^{h-2} α^{h-2} A white subtree has at least POSITION A red subtree has at least one

one white leaf node ∃ A SPECIAL QUERY ENTROPY NODE IN

CONCLUSION

FIGURE: This figure depicts a *label* $L_{\vec{F}}$ associated with a problem

instance Tree_₹

red leaf node



\exists MANY \vec{F} THAT REMAIN UNACCOUNTED WITHOUT SUCH A SPECIAL LABEL.

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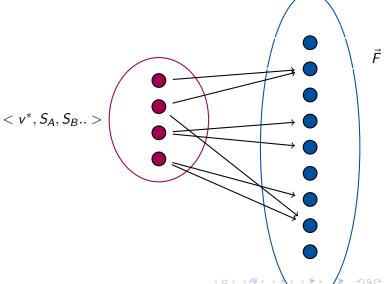
A SPECIAL QUERY

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SETS CONCLUSION

ODEN PRODUEME



OPEN PROBLEMS

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Open Problem

- More general time space tradeoffs for composition.
- Exponential lower bound for boolean semantic NBPs for some problem in P.
- super-quadratic lower bound for BPs via understanding composition fog where g is element distinctness.

Thank You!

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