[Minahan-Volkovich]'s proof

$$L_i(y) = \prod_{j \neq i} (y - \alpha_j)$$

$$\Phi(f(x_1, \ldots, x_n)) = f(zL_1(y), \ldots, zL_n(y)).$$

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Theorem ([Minahan-Volkovich])

If f is a non-constant ROF, then $\Phi(f)$ is a non-constant polynomial.

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Need to figure out why this holds when $f = P + Q$, a sum of two *variable disjoint* polynomials.

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Obvious GCD lemma

Suppose P is a homogeneous polynomial of degree d that depends on just the variables $V \subseteq X$. Then,

$$\Phi(P(x_1, \ldots, x_n)) = z^d \cdot \prod_{x_i \notin V} (y - \alpha_i)^d \cdot P'(y),$$

where $\deg P'(y) \leq d\,(|V| - 1)$.

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Proof.

Duh! $\square$

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$$P + Q$$

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$$P_0 + \cdots + P_a \quad + \quad Q_0 + \cdots + Q_b$$

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$$\Phi'(P_0) + \cdots + z^a \Phi'(P_a) \quad + \quad \Phi'(Q_0) + \cdots + z^b \Phi'(Q_b)$$

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$$z^d \left(\Phi'(P_d) + \Phi'(Q_d) \right)$$

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$$z^d \left(\Phi'(P_d) + \Phi'(Q_d) \right)$$

$$= z^d \prod_{x_i \notin V \cup W} (y - \alpha_i)^d \left(P_d'(y) \prod_{x_i \in W} (y - \alpha_i)^d + Q_d'(y) \prod_{x_i \in V} (y - \alpha_i)^d \right)$$

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But $\deg P'_d(y) \leq d(|V| - 1)$. □