Stochasticity in Algorithmic Statistics for Polynomial Time

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A black box that samples from an unknown probability distribution

A general question: Given the black box's output $x$ and a distribution $\mu$, is it plausible that the black box samples from $\mu$?

Example: Let $x = 101100101110100101010000101100101110100101010000$ and let $\mu$ be the uniform distribution over strings of length $n = |x|$.

Is it plausible that the black box samples from $\mu$?

An answer: No, since $x$ is a square ($x = uu$) and the probability of being a square is negligible ($2^{-n}$).
A black box that samples from an unknown probability distribution

\[
\rightarrow x = 100010101 \ldots 1^n
\]
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\[ \rightarrow x = 100010101110100101000101100101110100101010001 \ldots 1 \]

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Definition (Kolmogorov)

A probability distribution $\mu$ is an acceptable explanation for $x$ if the \textit{randomness deficiency} of $x$ wrt $\mu$,

$$- \log_2 \mu(x) - C(x|\mu)$$

is negligible.
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\[- \log_2 \mu(x) - C(x|\mu)\]

is negligible.

\textbf{Majority Principle}: for all $\mu$, if $x$ is sampled from $\mu$, then the probability of having 

\[- \log_2 \mu(x) - C(x|\mu) > \beta\]

is less than $2^{-\beta}$.
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**Proposition**

$- \log \mu(x) - C(x|\mu)$ is large if and only if there is a simple $T \ni x$ (that is, $T$ is enumerated by a short program) with negligible $\mu(T)$. 
Back to our example:

\[ x = 10110010111010010101000101100101110100101010000 \]
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If \( \mu \) is the uniform distribution over \( n \)-bit strings, then
\[
- \log \mu(x) - C(x|\mu) \approx n - \frac{n}{2} = \frac{n}{2};
\]

Another example: Let \( x \) be an arbitrary \( n \)-bit string, let \( \mu \) be concentrated on \( x \), i.e., \( \mu(x) = 1 \). Then \( \mu \) is acceptable for \( x \), since
\[
- \log \mu(x) - C(x|\mu) \approx 0 - 0 = 0.
\]
Back to our example:
\[ x = 1011001011101010000101100101110100101010000 \]

- If \( \mu \) is the uniform distribution over \( n \)-bit strings, then
  \[ -\log \mu(x) - C(x|\mu) \approx n - n/2 = n/2; \]
- If \( \mu \) is the uniform distribution over all \( n \)-bit squares, then
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The goal:

given \( x \), find a simple (\( C(\mu) \approx 0 \)) acceptable explanation \( \mu \) for \( x \).
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Theorem (A. Shen 1983)

This goal is not always achievable (there are non-stochastic strings).
Now we care about computation time!

Question: How do we define acceptable explanations? Why not say that time-bounded version of Kolmogorov's randomness deficiency $-\log \mu(x) - C_t(x|\mu)$ is small?

Answer: For polynomial time bounded computations, we cannot prove that randomness deficiency is small if and only if there is no simple refutation set. We will define acceptability using refutation sets.

Back to our example:

$x = 101100101110100101010000101100101110100101010000$, $\mu$ is the uniform distribution over strings of length $|x| = |x|$. We refute $\mu$, since $x$ falls into a simple set $T \ni x$ having negligible $\mu(T)$. Notice that $T$ can be recognized by a short program in a short (polynomial) time.
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Algorithmic Statistics with time bounds: acceptable explanations

Definition (informal)

$\mu$ is an acceptable explanation for $x$ if there is no $T \ni x$ with negligible $\mu(T)$ which is recognizable by a short program in a short time.

Definition (formal)

$\mu$ is a $(t, \alpha, \varepsilon)$-acceptable explanation for $x$ if for all $T \ni x$ with $CD_t(T) < \alpha$, we have $\mu(T) \geq \varepsilon$.

Majority principle: if $\varepsilon \ll 2^{-\alpha}$, then the $\mu$-probability of the event $\mu$ is not $(t, \alpha, \varepsilon)$-acceptable explanation for $x$ is negligible (the probability of this event is smaller than $\varepsilon 2^{-\alpha}$).
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**Definition (informal)**

\( \mu \) is an *acceptable explanation* for \( x \) if there is no \( T \ni x \) with negligible \( \mu(T) \) which is recognizable by a short program in a short time.

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**Majority principle:** If \( \varepsilon \ll 2^{-\alpha} \), then the \( \mu \)-probability of the event

\[ \mu \text{ is not } (t, \alpha, \varepsilon)\text{-acceptable explanation for } x \]

is negligible (the probability of this event is smaller than \( \varepsilon 2^\alpha \)).
Example

\(x\) is an arbitrary string and \(\mu\) is concentrated on \(x\). Then \(\mu\) is \((\ast, \ast, 1)\)-acceptable for \(x\).
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Goal: Given $x$ find a simple acceptable explanation for $x$. 
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Then \( \mu \) is \((*,*,1)\)-acceptable for \( x \).

**Goal:** Given \( x \) find a simple acceptable explanation for \( x \).

**Definition (informal)**

A distribution \( \mu \) is simple if there is a fast sampler with a short program for \( \mu \).
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$\mu$ is $(t', \alpha')$-simple if there is a sampler for $\mu$ with program of length less than $\alpha'$ and running time less than $t'$. 
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Remark

For one result we will need that $\mu$ be computed rather than sampled in a short time.
The main result

We consider \((t', \alpha')\)-simple \((t, \alpha, \varepsilon)\)-acceptable explanations where

- \(\alpha \gg \alpha'\),
- \(t \gg t'\), and
- \(\varepsilon \ll 2^{-\alpha}\).
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We consider \((t', \alpha')\)-simple \((t, \alpha, \varepsilon)\)-acceptable explanations where \(\alpha', \alpha\) are \(O(\log n)\),
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We consider \((t', \alpha')\)-simple \((t, \alpha, \varepsilon)\)-acceptable explanations where \(\alpha', \alpha \in O(\log n)\), \(t', t, 1/\varepsilon\) are polynomial in \(n\), and \(\alpha \gg \alpha', t \gg t', \text{ and } \varepsilon \ll 2^{-\alpha}\).

Conjecture (informal)

There are strings \(x\) that have no simple acceptable explanations (\textit{non-stochastic} strings).
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There are strings \(x\) that have no simple acceptable explanations (non-stochastic strings).

**Conjecture (formal)**

For all \(c\) there is \(d\) such that for infinitely many \(n\) there is a \(n\)-bit string \(x\) without \((n^c, c \log n)\)-simple \((n^d, d \log n, n^{-c})\)-acceptable explanations.

Theorem

If \(\text{NE} \neq \text{RE}\), then the Conjecture holds (and, moreover, the Conjecture holds for a constant \(d\), which does not depend on \(c\)).
The main result

We consider \((t', \alpha')\)-simple \((t, \alpha, \varepsilon)\)-acceptable explanations where \(\alpha', \alpha \in O(\log n)\), \(t', t, 1/\varepsilon\) are polynomial in \(n\), and \(\alpha \gg \alpha', t \gg t', \text{and} \varepsilon \ll 2^{-\alpha}\).

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If \(NE \neq RE\), then the Conjecture holds (and, moreover, the Conjecture holds for a constant \(d\), which does not depend on \(c\)).
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For all $c$ there is $d$ such that for infinitely many $n$ there is a $n$-bit string $x$ without $(n^c, c \log n)$-simple $(n^d, d \log n, n^{-c})$-acceptable explanations.

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### Theorem

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*If the Conjecture holds for a constant $d$ (not depending on $c$), then $P \neq \text{PSPACE}$.*
Other results

Conjecture (formal)
For all $c$ there is $d$ such that for infinitely many $n$ there is a $n$-bit string $x$ without $(n^c, c \log n)$-simple $(n^d, d \log n, n^{-c})$-acceptable explanations.

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If $\text{NE} \neq \text{RE}$, then the Conjecture holds (and, moreover, the Conjecture holds for a constant $d$, which does not depend on $c$).

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If the Conjecture holds for a constant $d$ (not depending on $c$), then $P \neq \text{PSPACE}$.

Theorem
If $P = \text{PSPACE}$, then the Conjecture holds. (Moreover, the Conjecture holds unconditionally for space restrictions in place of time restrictions.)
A set $T$ of strings is called *elusive* if $T \in \mathbb{P}$, however for all $c$ there are infinitely many $n$ such that $T^n \neq \emptyset$ but for any randomized machine with program of length $c \log n$ running in time $n^c$ we have $\Pr[M's \text{ output falls in } T^n] < n^{-c}$
The techniques

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**The sketch of the proof.**

$\text{NE} \neq \text{RE} \Rightarrow$ There exists an elusive set $\Rightarrow$ The Conjecture
A gap between Kolmogorov and distinguishing complexities

**Theorem (informal)**

*If there exists an elusive set, then there are strings $x$ with*

$$CD^{\text{poly}(n)}(x|r) \ll C^{\text{poly}(n)}(x|r)$$

*for 99% of $r$’s of length $\text{poly}(n)$.*
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### Theorem (formal)

*If there exists an elusive set, then for some $d$ for all $c$ there are infinitely many strings $x$ with*

$$CD^{\lceil x \rceil^d}(x|r) \ll C^{\lceil x \rceil^c}(x|r) - c \log \lceil x \rceil$$

*for 99% of $r$’s of length $n^d$.***
Other approaches do define acceptable explanations:

**Plausible explanations**

**Definition**

\[ \mu \text{ is a } (t, \varepsilon )\text{-plausible explanation for } x \text{ if for all } T \ni x \text{ we have } \mu (T) \geq \varepsilon 2^{-CD^t(T)}. \]
Other approaches do define acceptable explanations: Plausible explanations

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\( \mu \) is a \((t, \varepsilon)\)-plausible explanation for \( x \) if for all \( T \ni x \) we have \( \mu(T) \geq \varepsilon 2^{-CD_t(T)} \).

**Theorem**

Assume that there exists a PRNG \( G : \{0,1\}^n \rightarrow \{0,1\}^{2n} \). Then for all \( c \) for all sufficiently large \( n \) for 99\% of strings \( s \) of length \( n \) the uniform distribution is an \((n^c, c \log n, n^{-c}/200)\)-acceptable hypothesis for \( G_n(s) \).
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Assume that there exists a PRNG \( G : \{0,1\}^n \rightarrow \{0,1\}^{2n} \). Then for all \( c \) for all sufficiently large \( n \) for 99% of strings \( s \) of length \( n \) the uniform distribution is an \((n^c, c \log n, n^{-c}/200)\)-acceptable hypothesis for \( G_n(s) \).

On the other hand, the set \( T = \{x\} \) proves that the uniform distribution is not \((\text{poly}(n), 2^{-n})\)-plausible for \( x \), as the fraction of \( T = \{x\} \) among all \( 2n \)-bit strings is \( 2^{-2n} \) and \( \text{CD}^{\text{poly}(n)}(T) \leq n \).
Other approaches do define acceptable explanations: optimal explanations

**Definition**

\( \mu \) is a \((t, \varepsilon)\)-optimal explanation for \( x \) if \( \mu(x) \geq \varepsilon 2^{-C^t(x)} \).
Other approaches do define acceptable explanations: optimal explanations

**Definition**

μ is a \((t, \varepsilon)-optimal\) explanation for \(x\) if \(\mu(x) \geq \varepsilon 2^{-C^t(x)}\).

Relations between acceptability, plausibility and optimality
Other approaches do define acceptable explanations: optimal explanations

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\((t, \varepsilon)\)-plausible \( \Rightarrow \) \((t, \alpha, \varepsilon 2^{-\alpha})\)-acceptable for all \( \alpha \).
Other approaches do define acceptable explanations: optimal explanations

**Definition**

$\mu$ is a $(t, \varepsilon)$-optimal explanation for $x$ if $\mu(x) \geq \varepsilon 2^{-C^t(x)}$.

**Relations between acceptability, plausibility and optimality**

$(t, \varepsilon)$-plausible $\Rightarrow$ $(t, \alpha, \varepsilon 2^{-\alpha})$-acceptable for all $\alpha$.

$(t, \varepsilon)$-plausible $\Rightarrow$ $(t, \varepsilon)$-optimal (let $T = \{x\}$ in the definition of plausibility).
Other approaches do define acceptable explanations: optimal explanations

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\((t, \varepsilon)\)-plausible \( \Rightarrow \) \((t, \varepsilon)\)-optimal (let \( T = \{x\} \) in the definition of plausibility).

**Theorem (informal)**

(1) If \( CD^{\text{poly}}(x) \ll C^{\text{poly}}(x) \), then \( x \) has no simple plausible explanations (under the assumption that \( \text{Time}(2^{O(n)}) \not\subset \text{Space}(2^{o(n)}) \) for almost all \( n \)).
Other approaches do define acceptable explanations: optimal explanations

**Definition**

\( \mu \) is a \((t, \varepsilon)\)-optimal explanation for \( x \) if \( \mu(x) \geq \varepsilon^{2^{-C_t(x)}} \).

**Relations between acceptability, plausibility and optimality**

\((t, \varepsilon)\)-plausible \(\Rightarrow\) \((t, \alpha, \varepsilon^{2^{-\alpha}})\)-acceptable for all \(\alpha\).

\((t, \varepsilon)\)-plausible \(\Rightarrow\) \((t, \varepsilon)\)-optimal (let \( T = \{x\} \) in the definition of plausibility).

**Theorem (informal)**

1. If \( CD^{poly}(x) \ll C^{poly}(x) \), then \( x \) has no simple plausible explanations (under the assumption that \( Time(2^{O(n)}) \not\subset Space(2^{o(n)}) \) for almost all \( n \)).

2. If \( CD^{poly}(x) \approx C^{poly}(x) \), then every simple optimal explanation is plausible (under the assumption that \( Time(2^{O(n)}) \not\subset Size(2^{o(n)}) \) for almost all \( n \)).
Open questions

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Under which assumptions there exist non-stochastic strings with polynomial bounds for time and linear bounds for program length.
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Under which assumptions there exist strings that do not possess simple optimal hypotheses?
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Question
How acceptability is related to optimality for strings $x$ with $\text{CD}^{\text{poly}}(x) \ll C^{\text{poly}}(x)$?
Thank you!